Soft Computing Fusion with Applications

www.journal-scfa.com

Soft. Comput. Fusion. Appl. Vol. 1, No. 1 (2024) 19–26.

Paper Type: Original Article

A New Approach for Solving Fuzzy Cooperative Continuous Static Games

Seyyed Ahmad Edalatpanah [1], Hamiden Abd El- Wahed Khalifa [2], Robert S. Keyser[3, *]

[1] Applied Mathematics Department, Ayandegan Institute of Higher Education, Tonekabon, Iran; s.a.edalatpanah@aihe.ac.ir.
[2] Operations Research Department, Faculty of Graduate Studies for Statistical Research, Cairo University, Giza, Egypt; hamiden@cu.edu.eg.
[2] Mathematics Department, College of Science and Arts, Al- Badaya, Qassim University, Saudi Arabia; Ha.Ahmed@qu.edu.sa.
[3] Southern Polytechnic College of Engineering and Engineering Technology, Kennesaw State University, Marietta, Georgia 30060, USA; rkeyser@kennesaw.edu.

Citation:

Received: 08 April 2023 *Revised: July 2023* *Accepted:27/10/2023*	Edalatpanah, S. A., Khalifa, H. A. E. W., & Keyser, R. S. (2024). A new approach for solving fuzzy cooperative continuous static games. Soft Computing Fusion with Applications, *1*(1), 19-26.

Abstract

This study explores Cooperative Continuous Static Games (CCSGs) characterized by fuzzy cost functions that are piecewise quadratic. It introduces an effective approximation technique, specifically the close interval approximation for Piecewise Quadratic Fuzzy Numbers (PQFNs). The paper employs the weighted Tchebycheff method to derive an optimal compromise solution and establishes the corresponding stability set of the first kind. Additionally, a numerical example is provided to illustrate the effectiveness of the proposed method in computational terms.

Keywords: Cooperative continuous static games, Close interval approximation, Piecewise quadratic fuzzy numbers, α-efficient solution, Weighted Tchebycheff method, Optimal compromise solution, Stability.

1 | Introduction

The possibility of competition among the system controllers is called "players," and the optimization problem under consideration is, therefore, termed a " game". Each player in the game controls a specified subset of the system parameters (called his/her control vector) and seeks to minimize his/her own scaler cost criterion subject to specified constraints. Game theory applications may be found in economics, engineering, biology, etc. Three major classes of games are:

– *Matrix games.*
– *Continuous static games.*

Corresponding Author: rkeyser@kennesaw.edu

– *Differential games.*

The three basic solution concepts for the games Vincent et al. [1] are:

I. Nash equilibrium solutions.

II. Min-max solutions.

III. Pareto minimal solutions.

In many scientific areas, such as system analysis and operator research, a model must be set up-using data, which is only approximately known. Fuzzy sets theory, introduced by Zadeh [2], makes this possible. Fuzzy numerical data can be represented by employing fuzzy subsets of the real line, known as fuzzy numbers. Dubois and Prade [3] extended the use of algebraic operations on real numbers to fuzzy numbers by the use of a fuzzification principle. Despite vast decision-making experience, the decision-maker cannot articulate the goals precisely. Decision-making in a fuzzy environment, developed by Bellman and Zadeh [4], improved and is a great help in managing decision problems. Zimmermann [5] proposed the fuzzy set theory and its applications.

Orlovski [6] studied Multi-Objective Nonlinear Programming (MONLP) problems with fuzzy parameters. Osman and El-Banna [7] introduced the stability of fuzzy MONLP. Many pieces of research have been introduced in CCSGs Khalifa and Zeineldin [8]; Khalifa [9]; Donahue et al. [10]; Zhou et al.[11].

In his earlier work, Osman [12] , [13] analyzed the notions of the solvability set, stability set of the first kind, and stability set of the second kind for parametric convex nonlinear programming.

This paper introduces fuzzy Cooperative Continuous Static Games (CCSGs). The weighted Tchebycheff method is applied to obtain the optimal compromise solution.

The remainder of the paper is organized as follows: Section 2 introduces some preliminaries needed in this paper. Section 3 formulated the mathematical model for the continuous cooperative static games. Section 4 proposed a solution approach for obtaining an optimal compromise solution. Section 5 gives a numerical example for illustration. Finally, some concluding remarks are reported in section 6.

2 | Preliminaries

To discuss the problem easily, it recalls basic rules and findings related to fuzzy numbers, Piecewise Quadratic Fuzzy Numbers (PQFNs), close interval approximation, and its arithmetic operations.

Definition 1 ([2]). Fuzzy number: a fuzzy number $\widetilde{A}$ is a fuzzy set with a membership function defined as $\pi_{\widetilde{A}}(x): \Re \to [0,1]$, and satisfies:

I. $\widetilde{A}$ is fuzzy convex, i.e., $\pi_{\widetilde{A}}(\delta x + (1-\delta) y) \geq \min\{\pi_{\widetilde{A}}(x), \pi_{\widetilde{A}}(y)\};$ for all $x, y \in \Re; 0 \leq \delta \leq 1$.

II. $\widetilde{A}$ is normal, i.e., $\exists\, x_0 \in \Re$ for which $\pi_{\widetilde{A}}(x_0) = 1$.

III. $\text{Supp}\,(\tilde{A}) = \{x \in \Re: \pi_{\tilde{A}}(x) > 0\}$ is the support of $\widetilde{A}$.

IV. $\pi_{\widetilde{A}}(x)$ is an upper semi-continuous (i. e., for each $\alpha \in (0,1)$, the α-cut set $\widetilde{A}_\alpha = \{x \in \Re: \pi_{\widetilde{A}} \geq \alpha\}$ is closed.

Definition 2 ([14]). A PQFN is denoted by $\widetilde{A}_{PQ} = (a_1, a_2, a_3, a_4, a_5)$, where $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$ are real numbers and is defined by if its membership function $\mu_{\tilde{a}_{PQ}}$ is given by (*Fig. 1*).

$$\mu_{\tilde{A}_{PQ}} = \begin{cases} 0, x < a_1, \\ \frac{1}{2}\frac{1}{(a_2-a_1)^2}(x-a_1)^2, \quad a_1 \le x \le a_2, \\ \frac{1}{2}\frac{1}{(a_3-a_2)^2}(x-a_3)^2+1, \quad a_2 \le x \le a_3, \\ \frac{1}{2}\frac{1}{(a_4-a_3)^2}(x-a_3)^2+1, \quad a_3 \le x \le a_4, \\ \frac{1}{2}\frac{1}{(a_5-a_4)^2}(x-a_5)^2, a_4 \le x \le a_5, \\ 0, x > a_5. \end{cases}$$

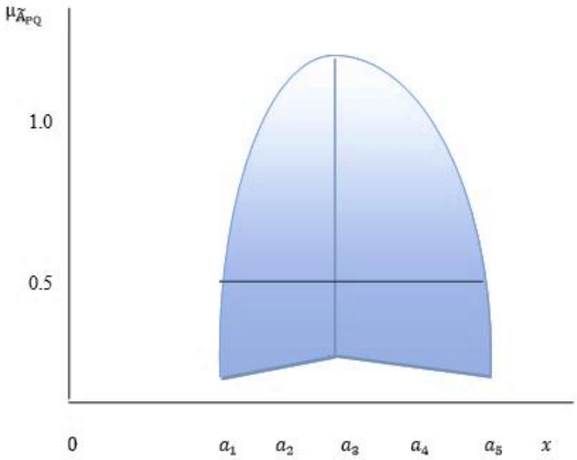

Fig. 1. Graphical representation of a PQFN.

Definition 3 ([14]). Let $\tilde{A}_{PQ} = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B}_{PQ} = (b_1, b_2, b_3, b_4, b_5)$ be two PQFNs. The arithmetic operations on $\tilde{A}_{PQ}$ and $\tilde{B}_{PQ}$ are:

I. Addition: $\tilde{A}_{PQ}(+)\tilde{B}_{PQ} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5)$.

II. Subtraction: $\tilde{A}_{PQ}(-)\tilde{B}_{PQ} = (a_1 + b_5, a_2 + b_4, a_3 + b_3, a_4 + b_2, a_5 + b_1)$.

III. Scalar multiplication: $k\tilde{A}_{PQ} = \begin{cases} (k\,a_1, k\,a_2, k\,a_3, k\,a_4, k\,a_5), k > 0, \\ (k\,a_5, k\,a_4, k\,a_3, k\,a_2, k\,a_1), k < 0. \end{cases}$

Definition 4 ([14]). An interval approximation $[A] = [a_\alpha^-, a_\alpha^+]$ of a PQFN $\tilde{A}$ is called closed interval approximation if: $a_\alpha^- = \inf\{x \in \mathbb{R}: \mu_{\tilde{A}} \ge 0.5\}$, and $a_\alpha^+ = \sup\{x \in \mathbb{R}: \mu_{\tilde{A}} \ge 0.5\}$.

Definition 5 ([14]). Associated ordinary numbers [14]. If $[A] = [a_\alpha^-, a_\alpha^+]$ is the close interval approximation of PQFN, the Associated ordinary number of [A] is defined as $\hat{A} = \frac{a_\alpha^- + a_\alpha^+}{2}$.

Definition 6 ([14]). Let $[A] = [a_\alpha^-, a_\alpha^+]$, and $[B] = [b_\alpha^-, b_\alpha^+]$ be two interval approximations of PQFN. Then, the arithmetic operations are

I. Addition: $[A](+)[B] = [a_\alpha^- + b_\alpha^-, a_\alpha^+ + b_\alpha^+]$.

II. Subtraction: $[A](-)[B] = [a_\alpha^- - b_\alpha^+, a_\alpha^+ - b_\alpha^-]$.

III. Scalar multiplication: $\alpha\,[A] = \begin{cases} [\alpha\, a_\alpha^-, \alpha\, a_\alpha^+], \alpha > 0, \\ [\alpha\, a_\alpha^+, \alpha\, a_\alpha^-], \alpha < 0. \end{cases}$

IV. Multiplication: $[A](\times)[B] = \left[\frac{a_\alpha^+ b_\alpha^- + a_\alpha^- b_\alpha^+}{2}, \frac{a_\alpha^- b_\alpha^- + a_\alpha^+ b_\alpha^+}{2}\right]$.

V. Division: $[A](\div)[B]=\begin{cases}\left[2\left(\frac{a_\alpha^-}{b_\alpha^-+b_\alpha^+}\right),2\left(\frac{a_\alpha^+}{b_\alpha^-+b_\alpha^+}\right)\right],[B]>0,b_\alpha^-+b_\alpha^+\neq 0,\\ \left[2\left(\frac{a_\alpha^+}{b_\alpha^-+b_\alpha^+}\right),2\left(\frac{a_\alpha^-}{b_\alpha^-+b_\alpha^+}\right)\right],[B]<0,b_\alpha^-+b_\alpha^+\neq 0.\end{cases}$

VI. The order relations:

- $[A](\lesssim)[B]$ *if* $a_\alpha^- \le b_\alpha^-$ *and* $a_\alpha^+ \le b_\alpha^+$ *or* $a_\alpha^- + a_\alpha^+ \le b_\alpha^- + b_\alpha^+$.
- $[A]$ *is preferred to* $[B]$ *if and only if* $a_\alpha^- \ge b_\alpha^-, a_\alpha^+ \ge b_\alpha^+$.

It is noted that $P(\mathbb{R}) \subset F(\mathbb{R})$t, where $F(\mathbb{R})$, and $P(\mathbb{R})$ are the sets of all PQFNs and close in interval approximation of PQFN, respectively.

3 | Problem Formulation and Solution Concepts

Consider the following Fuzzy Cooperative Continuous Static Games (F-CCSG) with n-players having piecewise quadratic fuzzy parameters in the cost functions of the players. These players respectively have the costs.

F- CCSG

$$G_1(b,,\xi,\ \tilde{a}_1),G_2(b,\xi,\tilde{a}_1),\dots,G_m(b,\xi,\tilde{a}_m), \quad (1)$$

s.t.

$$g_j(b,\xi)=0, j=\overline{1,n}, \quad (2)$$

$$\xi\in\Omega=\{\xi\in\mathfrak{R}^s: h_l(b,\xi)\ge 0, l=\overline{1,r}\}. \quad (3)$$

Where, $G_i(b,\xi,\tilde{a}_i), ji=\overline{1,m}$ are convex functions on $\mathfrak{R}^n\times\mathfrak{R}^s$, $h_l(b,\xi), l=\overline{1,r}$ are concave functions on $\mathfrak{R}^n\times\mathfrak{R}^s$, $g_j(b,\xi), j=\overline{1,n}$ are convex functions on $\mathfrak{R}^n\times\mathfrak{R}^s$Assume that there exists a function $b=f(\xi)$. If the function $g_j(b,\xi)=0, j=\overline{1,n}$ are differentiable than the Jacobian $\left|\frac{\partial g_j(b,\xi)}{\partial b_q}\right|\neq 0, j; q=\overline{1,n}$ in the neighborhood of a solution point (b,ξ) to *Eq. (2)*, $b=f(\xi)$ is the solution to *Eq. (2)* generated by $\xi\in\Omega$; differentiability assumptions are not needed here for all the functions $G_i(b,\gamma,\tilde{a}_i), i=\overline{1,n}$ and $h_l(b,\xi)$, Ω is a regular and compact set. $\tilde{a}_i, i=\overline{1,m}$, represents a vector of PQFNs Jain [14]. Let $\tilde{a}_1,\tilde{a}_2,\dots,\tilde{a}_m$; $\mu_{\tilde{a}_1}(a_1),\mu_{\tilde{a}_2}(a_2)\dots,\mu_{\tilde{a}_m}(a_m)$ be the PQFNs in the F-CCSG problem with convex membership functions, respectively.

For a certain degree of α, the F-CCSG problem can be rewritten in the following fuzzy form Sakawa and Yano [15], [16]:

α- CCSG

$$G_1(b,\xi,a_1),G_2(b,\xi,a_2),\dots,G_n(b,\gamma,a_m), \quad (4)$$

s.t.

$$g_j(b,\xi)=0, j=1,2,\dots,n, \quad (5)$$

$$\Omega=\{\xi\in\mathfrak{R}^s: h_l(b,\xi)\ge 0, l=\overline{1,r}\}, \quad (6)$$

$$a_i\in L_\alpha(\tilde{a}_i), i=\overline{1,m}. \quad (7)$$

Definition 7 ([1]). Let $b=f(\xi)$ be the solution to *Eq. (5)* generated by $\xi\in\Omega$. A point $\xi^*\in\Omega$ is called an α-Pareto optimal solution to the α-CCSG problem, if and only if there does not exist $(\xi,a)\in\ \Omega\times L_\alpha(\tilde{a}_i)$ such that:

$G_i(f(\xi),\xi,a_i)\le G_i(f(\xi^*),\xi^*,\ a_i^*)$; for all $i=\overline{1,m}$ and $G_i(f(\xi),\xi,a_i)<G_i(f(\xi),\xi^*,\ a_i^*)$ for some $i\in\{1,2,\dots,m\}$, where a_i^* are called α-level minimal parameters.

From the α-Pareto optimal solution to the α-CCSG problem concept, one can show that a point $\xi^* \in \Omega$ is an α-Pareto minimal solution to the α-CCSG problem if and only if ξ^* is an α-Pareto minimal solution to the following α-multi-objective optimization problem

α- MOP

$$\min\left(\overline{G}_1(\xi, a_1), \overline{G}_2(\xi, a_2), \dots, \overline{G}_m(\xi, a_m)\right)^T, \quad (8)$$

s.t.

$$\Omega = \{\xi \in \Re^s : \overline{h}_l(b, \xi) \geq 0, l = \overline{1, r}\},$$

$$a_i \in L_\alpha(\tilde{a}_i), i = \overline{1, m}.$$

Where, $\overline{G}_i(\xi, a_i), i = \overline{1, m}$ are convex functions on $\Re^n \times \Re^t$ and $\overline{h}_l(\xi), l = 1, 2, \dots, r$ are concave functions on $\Re^s$ and $\overline{G}_i(\xi, a_i) = G_i(f(\xi), \xi, a_i), \overline{h}_l(\xi) = h_l(f(\xi), \xi)$. Assume that the α-MOP is to be stable Rockafellar [17]. Problem *Eq. (8)* will be solved by the weighting Tchebycheff problem

$$\min_{\xi \in \Omega a_i \in L_\alpha(\tilde{a}_i)} \max_{1 \leq i \leq m} \left\{w_i\left(\overline{G}_i(\xi, a_i) - \overline{G}_i(\xi^*, a_i^*)\right), a_i \in L_\alpha(\tilde{a}_i), i = \overline{1, m}\right\}, \quad (9)$$

or

$$\min\left\{\lambda : w_i\left(\overline{G}_i(\xi, a_i) - \overline{G}_i(\xi^*, a_i^*)\right) \leq \lambda, \xi \in \Omega, a_i \in L_\alpha(\tilde{a}_i), i = \overline{1, m}\right\}. \quad (10)$$

Where, $w_i \geq, i = \overline{1, m}$, and $\overline{G}_i(\xi^*, a_i^*), i = \overline{1, m}$ are the ideal targets. It is noted that the stability of α- MOP implies the stability of the *Eq. (10)*.

4 | Solution Procedure

The solution method is based on determining the α-best compromise solution within the close interval approximation of PQFNs and the minimum deviation from the $\overline{G}_i(\xi^*, a_i^*)$, where

$$\overline{G}_i(\xi^*, a_i^*) = \min_{\xi \in \Omega, a_i \in L_\alpha(\tilde{a}_i)} \overline{G}_i(\xi, a_i), i = \overline{1, m}.$$

Step 1. Calculate $\overline{G}_i^{min}$, and $\overline{G}_i^{max}$ (i.e., individual minimum and maximum) at $\alpha = 0$ and $\alpha = 1$, respectively.

Step 2. Compute the weight from the relation

$$w_i = \frac{\overline{G}_i^{max} - \overline{G}_i^{min}}{\sum_{i=1}^m \left(\overline{G}_i^{max} - \overline{G}_i^{min}\right)},$$

Step 3. Formulate and solve the following problem:

$$\min \lambda, \quad (11)$$

s.t.

$$W_i\left(\overline{G}_i(\xi, a_i) - \overline{G}_i(\xi^*, a_i^*)\right) \leq \lambda, i = \overline{1, m},$$

$$\xi \in \Omega, a_i = [(a_i)_\alpha^-, (a_i)_\alpha^+], i = \overline{1, m}.$$

Where, $W_i \geq 0, i = \overline{1, m}, \ \sum_{i=1}^m w_i = 1, [(a_{1i})_\alpha^-, (a_{2i})_\alpha^+] = L_\alpha(\tilde{a}_i), i = \overline{1, m}.$

Let (ξ°, a_i°) be the α- optimal compromise solution.

Step 4. Determine the stability set of the first kind $S(\xi^\circ, a_i^\circ)$.

Let $d = (d_1, d_2) \in \Re^{2m}$, where $d_1 = (d_{11}, \dots, d_{im})^T, d_2 = (d_{21}, \dots, d_{2m})^T$. Suppose that *Eq. (11)* is solvable for $(w^\circ, d^\circ) \in \Re^{3m}$ with a corresponding α-Pareto optimal solution (ξ°, a_i°) be given. $S(\xi^\circ, a_i^\circ)$ is determined by applying the following conditions:

$$\zeta_i^\circ(a_i^\circ - d_{2i}) = 0, i = \overline{1, m},$$
$$\eta_i^\circ(d_{1i} - a_i^\circ) = 0, i = \overline{1, m},$$
$$\zeta_i^\circ, \eta_i^\circ \geq 0, d_{1i}, d_{2i} \in \Re, [(a_{1i})_\alpha^-, (a_{2i})_\alpha^+] = L_\alpha(\tilde{a}_i), i = \overline{1, m}.$$

Example

Consider the following two-player game with

$$\overline{G}_1(\xi, \tilde{a}_1) = (\xi_1 - \tilde{a}_1)^2 + (\xi_2 - 1)^2$$

$$\overline{G}_2(\xi, \tilde{a}_2) = (\xi_1 - 1)^2 + \tilde{a}_2(\xi_2 - 2)^2$$

Where player 1 controls $\xi_1 \in \Re$, and player 2 controls $\xi_2 \in \Re$ with

$$\xi_1 - 4 \leq 0, \xi_2 - 4 \leq 0, -\xi_1 \leq 0, -\xi_2 \leq 0.$$

Let $\tilde{a}_1 = (1, 2, 3, 4, 5,)$ and $\tilde{a}_1 = (1, 3, 5, 9, 10)$ with the close interval approximation are $[(\tilde{a}_1)_\alpha] = [2, 4]$ and $[(\tilde{a}_2)_\alpha] = [3, 9]$.

Step 1. Solve

$$\min(\xi_1 - 1)^2 + (\xi_2 - 1)^2,$$

s.t.

$$\xi_1 - 4 \leq 0, \xi_2 - 4 \leq 0, -\xi_1 \leq 0, -\xi_2 \leq 0,$$
$$\mu_{\tilde{a}_1}(a_1) = 0, \mu_{\tilde{a}_2}(a_2) = 0.$$

Let $(\xi_1, \xi_2, a_1 = 1) = (1, 1, 1)$ with $\overline{G}_1^{min} = 0$.

Solve

$$\min(\xi_1 - 1)^2 + 10(\xi_2 - 2)^2,$$

s.t.

$$\xi_1 - 4 \leq 0, \xi_2 - 4 \leq 0, -\xi_1 \leq 0, -\xi_2 \leq 0,$$
$$\mu_{\tilde{a}_1}(a_1) = 0, \mu_{\tilde{a}_2}(a_2) = 0.$$

Let $(\xi_1, \xi_2, a_2 = 1) = (1, 2, 1)$ with $\overline{G}_2^{min} = 0$.

Solve

$$\max(\xi_1 - 3)^2 + (\xi_2 - 1)^2,$$

s.t.

$$\xi_1 - 4 \leq 0, \xi_2 - 4 \leq 0, -\xi_1 \leq 0, -\xi_2 \leq 0,$$
$$\mu_{\tilde{a}_1}(a_1) = 1, \mu_{\tilde{a}_2}(a_2) = 1.$$

Solve

$$\max(\xi_1 - 1)^2 + 5(\xi_2 - 2)^2,$$

s.t.

$$\xi_1 - 4 \leq 0, \xi_2 - 4 \leq 0, -\xi_1 \leq 0, -\xi_2 \leq 0,$$
$$\mu_{\tilde{a}_1}(a_1) = 1, \mu_{\tilde{a}_2}(a_2) = 1.$$

Let $(\xi_1, \xi_2, a_2 = 5) = (4, 0, 5)$ with $\overline{G}_2^{max} = 29$.

Step 2. $w_1 = \frac{\overline{G}_1^{max} - \overline{G}_1^{min}}{\left(\overline{G}_1^{max} - \overline{G}_1^{min}\right) + \left(\overline{G}_2^{max} - \overline{G}_2^{min}\right)} = 0.383$ and $w_2 = \frac{\overline{G}_2^{max} - \overline{G}_2^{min}}{\left(\overline{G}_1^{max} - \overline{G}_1^{min}\right) + \left(\overline{G}_2^{max} - \overline{G}_2^{min}\right)} = 0.617$.

Step 3. Solve

$$
\begin{aligned}
&\text{Min}\, \lambda, \\
&\text{s.t.} \\
&(\xi_1 - a_1)^2 + (\xi_2 - 1)^2 - \frac{47}{18}\lambda \leq 0, \\
&(\xi_1 - 1)^2 + a_2(\xi_2 - 2)^2 - \frac{47}{29}\lambda \leq 0, \\
&2 \leq a_1 \leq 4, = [2, 4], \text{and } 3 \leq a_2 \leq 9, \\
&\xi_1 - 4 \leq 0, \xi_2 - 4 \leq 0, -\xi_1 \leq 0, -\xi_2 \leq 0,
\end{aligned}
$$

Yields, $\xi_1^\circ = 1.440665$, $\xi_2^\circ = 1, a_1^\circ = 2$, $a_2^\circ = 3$ and $\lambda^\circ = 0.1198169$.

Step 4. Determine $S\,(1.440665, 1, 2, 3)$ by applying the following conditions:

$$\zeta_1^\circ(2 - d_{21}) = 0, \zeta_2^\circ(3 - d_{22}) = 0,$$

$$\eta_1^\circ(d_{11} - 2) = 0, \;\; \eta_2^\circ(3 - d_{12}) = 0,$$

$$\zeta_1^\circ, \zeta_2^\circ;\; \eta_1^\circ, \eta_2^\circ \geq 0, [c_{1i}, c_{2i}] = L_\alpha(\tilde{a}_i), i = 1, 2.$$

We have $J_{1k}; J_{2k} \subseteq \{1, 2\}$, for $J_{11} = \{1\}, \zeta_1^\circ, > 0\; \zeta_2^\circ = 0$.

For $J_{21} = \{2\}, \eta_1^\circ = 0, \eta_2^\circ = 0$, then

$$S_{J_{11},J_{21}}(1.440665, 1, 2, 3) = \begin{Bmatrix} (d_1,\ d_2) \in \Re^4: \\ d_{21} = 2, \\ d_{22} \geq 3, \\ d_{11} \leq 2, \\ d_{12} = 3, \end{Bmatrix}.$$

For $J_{12} = \{2\}, \zeta_1^\circ = 0, \zeta_2^\circ > 0$. For $J_{22} = \{1\}, \eta_1^\circ > 0, \eta_2^\circ = 0$, then

$$S_{J_{12},J_{22}}(1.440665, 1, 2, 3) = \begin{Bmatrix} (d_1,\ d_2) \in \Re^4: \\ d_{21} \geq 2, \\ d_{22} = 3, \\ d_{11} = 2, \\ d_{12} \leq 3, \end{Bmatrix}.$$

For $J_{13} = \{1{,}2\}, \zeta_1^\circ > 0, \zeta_2^\circ > 0$. For $J_{23} = \emptyset, \eta_1^\circ = 0, \eta_2^\circ = 0$, then

$$S_{J_{13},J_{23}}(1.440665, 1, 2, 3) = \begin{Bmatrix} (d_1,\ d_2) \in \Re^4: \\ d_{21} = 2, \\ d_{22} = 3, \\ d_{11} \leq 2, \\ d_{12} \leq 3, \end{Bmatrix}.$$

For $J_{14} = \emptyset, \zeta_1^\circ = 0, \zeta_2^\circ = 0$. For $J_{24} = \{1, 2\}, \eta_1^\circ > 0, \eta_2^\circ > 0$, then

$$S_{J_{14},J_{24}}(1.440665, 1, 2, 3) = \begin{Bmatrix} (d_1,\ d_2) \in \Re^4: \\ d_{21} \geq 2, \\ d_{22} \geq 3, \\ d_{11} = 2, \\ d_{12} = 3, \end{Bmatrix}.$$

Hence,

$$S(1.440665, 1, 2, 3) = \bigcup_{k=1}^{4} S_{J_{1k},J_{2k}}(1.440665, 1, 2, 3).$$

6 | Conclusions

This paper studies CCSG with PQFNs. The weighted Tchebycheff method has been applied to obtain the α-optimal compromise solution; hence, the stability set of the first kind corresponding to the obtained solution has been determined. The advantage of the approach is that it enables the decision-maker to have a satisfactory solution.

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