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Soft Intersection Almost Bi-quasi Ideals of Semigroups

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Abstract

The concept of bi-quasi-ideal generalizes the notions of bi-ideals and quasi-ideals in a semigroup; similarly, the soft intersection bi-quasi-ideal generalizes the concepts of soft intersection bi-ideals and soft intersection quasi-ideals in a semigroup. In this paper, we introduce the concept of soft intersection almost bi-quasi ideal and its generalized concept, soft intersection weakly almost bi-quasi ideals, in a semigroup. In contrast to the soft intersection ideal theory, we demonstrate that every soft intersection almost bi-quasi ideal is also a soft intersection almost ideal and a soft intersection almost bi-ideal. Additionally, we show that every idempotent soft intersection almost bi-quasi ideal is a soft intersection almost subsemigroup, a soft intersection almost weak interior ideal, a soft intersection almost tri-ideal, and a soft intersection almost tri-bi-ideal. Furthermore, we derive several interesting relationships regarding minimality, primeness, semiprimeness, and strong primeness between almost bi-quasi ideals and soft intersection almost bi-quasi ideals with the proven theorem stating that if a nonempty set A is an almost bi-quasi ideal, then its soft characteristic function is also a soft intersection almost bi-quasi ideal, and vice versa.

Keywords: Soft set, Semigroup, Bi-quasi ideals, Soft intersection (almost) bi-quasi ideals.

1 | Introduction

Semigroups are crucial in many branches of mathematics because they provide the abstract algebraic foundation for memoryless systems, which restart on each iteration. Semigroups were first explored formally in the early 1900s. Semigroups are fundamental models for linear time-invariant systems in practical mathematics. Studying finite semigroups is crucial to theoretical computer science, as they are naturally related to finite automata. In addition, semigroups and Markov processes are connected in probability theory.

Ideals are required to comprehend algebraic structures and their uses. In 1952, bi-ideals for semigroups were initially presented by Good and Hughes [1]. The concept of quasi-ideals was introduced by Steinfeld [2]

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initially for semigroups and then extended to rings. Generalizing ideals in algebraic structures has been a major study area for many mathematicians.

In 1980, Grosek and Satko [3] originally introduced the idea of almost left, right, and two-sided ideals of semigroups. Bogdanovic [4] generalized the notion of bi-ideals to almost bi-ideals in semigroups later, in 1981. In 2018, Wattanatripop et al. [5] defined almost quasi-ideals by combining the ideas of almost ideals and quasi-ideals of semigroups, expanding on the concepts of almost ideals and interior ideals of semigroups, and examining their characteristics. Kaopusek et al. [6] proposed almost interior ideals and weakly almost interior ideals of semigroups in 2020. Iampan [7] in 2021, Chinram and Nakkhasen [8] in 2022, Gaketem [9] in 2022, and Gaketem and Chinram [10] in 2023, respectively, subsequently introduced almost subsemigroups, almost bi-quasi-interior ideals, almost bi-interior ideals, and almost bi-quasi ideals of semigroups. Additionally, several almost fuzzy semigroup ideal types were investigated in [5], [7–12].

In 1999, Molodtsov [13] was the first to propose the idea of the soft set as a way to model uncertainty; this idea has subsequently drawn attention from a variety of fields. In [14–23], the fundamental operations of soft sets were examined. Çağman and Enginoğlu [24] modified the idea and presented soft intersection groups [25], which sparked studies on a number of soft algebraic systems. As thoroughly reviewed in [26], [27], soft sets were also conveyed to semigroup theory with the concepts of semigroups with soft intersection left, right, and two-sided ideals, quasi-ideals, interior ideals, and generalized bi-ideals. Different semigroups were categorized by Sezgin and Orbay [28] using soft intersection substructures.

Further research was done on a range of soft algebraic structures in [29–38]. Rao [39–42] has developed several new semigroup types, including bi-interior ideals, bi-quasi-interior ideals, bi-quasi ideals, quasi-interior ideals, and weak interior ideals, extensions of existing ideals. Furthermore, Baupradist et al. [43] proposed the idea of essential ideals in semigroups.

Rao introduced the bi-quasi ideal of semigroups [41] as a generalization of bi-ideal and quasi-ideal. In contrast, the soft intersection bi-quasi ideal of semigroups was proposed in [44] to generalize the soft intersection bi-ideal and soft intersection quasi-ideal. In [10], almost bi-quasi ideals are introduced as a further generalization of bi-quasi ideals defined in [41]. This study proposes the concept of soft intersection almost bi-quasi ideals and its generalization soft intersection weakly almost bi-quasi ideals of semigroups. Moreover, in contrast to soft intersection semigroup theory, our results show that every soft intersection almost bi-quasi ideal is also a soft intersection almost ideal and a soft intersection almost bi-ideal.

Furthermore, we show that an idempotent soft intersection almost bi-quasi ideal is a soft intersection almost subsemigroup, a soft intersection almost weak interior ideal, a soft intersection almost tri-ideal, and a soft intersection almost tri-bi-ideal. We note that a semigroup may be constructed by soft intersection almost bi-quasi ideals of a semigroup under the binary operation of soft union but not under the soft intersection operation. Also, by deriving that if a nonempty set A is almost bi-quasi ideal, then its soft characteristic function is also a soft intersection almost bi-quasi ideal, and vice versa, we establish the relationship between a semigroup's soft intersection almost bi-quasi ideal and almost bi-quasi ideal as regards minimality, primeness, semiprimeness, and strongly primeness.

2 | Preliminaries

This section reviews several fundamental notions related to semigroups and soft sets.

Definition 1. Let U be the universal set, E be the parameter set, $P(U)$ be the power set of U, and $K \subseteq E$. A soft set f_K over U is a set-valued function such that $f_K: E \to P(U)$ such that for all $x \notin K$, $f_K(x) = \emptyset$. A soft set over U can be represented by the set of ordered pairs [13], [24].

$$f_K = \{(x, f_K(x)): x \in E, f_K(x) \in P(U)\}.$$

Throughout this paper, the set of all the soft sets over U is designated by $S_E(U)$.

Definition 2. Let $f_A \in S_E(U)$. If $f_A(x) = \emptyset$ for all $x \in E$, then f_A is called a null soft set and denoted by $\emptyset_E$. If $f_A(x) = U$ for all $x \in E$, then f_A is called an absolute soft set and is denoted by U_E [24].

Definition 3. Let $f_A, f_B \in S_E(U)$. If $f_A(x) \subseteq f_B(x)$ for all $x \in E$, then f_A is a soft subset of f_B and denoted by $f_A \tilde{\subseteq} f_B$. If $f_A(x) = f_B(x)$ for all $x \in E$, then f_A is called soft equal to f_B and denoted by $f_A = f_B$ [24].

Definition 4. Let f_A, $f_B \in S_E(U)$. The union of f_A and f_B is the soft set $f_A \tilde{\cup} f_B$, where $(f_A \tilde{\cup} f_B)(x) = f_A(x) \cup f_B(x)$, for all $x \in E$. The intersection of f_A and f_B is the soft set $f_A \tilde{\cap} f_B$, where $(f_A \tilde{\cap} f_B)(x) = f_A(x) \cap f_B(x)$, for all $x \in E$ [24].

Definition 5. For a soft set f_A, the support of f_A is defined by [45]:

$$\mathrm{supp}(f_A) = \{x \in A: f_A(x) \neq \emptyset\}$$

It is obvious that a soft set with an empty support is a null soft set; otherwise, the soft set is non-null.

Note 1. If $f_A \tilde{\subseteq} f_B$, then $\mathrm{supp}(f_A) \subseteq \mathrm{supp}(f_B)$ [46].

A semigroup S is a nonempty set with an associative binary operation, and throughout this paper, S stands for a semigroup, and all the soft sets are the elements of $S_S(U)$ unless otherwise specified.

Definition 6. A nonempty subset F of S is called

I. A left (right) bi-quasi ideal of S if $SF \cap FSF \subseteq F$ ($FS \cap FSF \subseteq F$), and a bi-quasi ideal of *S* if F is both a left bi-quasi ideal of S and right bi-quasi ideal of S [41].

II. An almost left (right) ideal of S if $wF \cap F \neq \emptyset$ ($Fw \cap F \neq \emptyset$), for all $w \in S$, and an almost ideal of S if F is both an almost left ideal of S and an almost right ideal of S [3].

III. An almost bi-ideal of S if $FwF \cap F \neq \emptyset$, for all $w \in S$ [4].

IV. An almost left (right) bi-quasi ideal of S if $(wF \cap FxF) \cap F \neq \emptyset$ $\big((Fw \cap FxF) \cap F \neq \emptyset\big)$, for all $w, x \in S$, and an almost bi-quasi ideal (briefly almost BQ-ideal) of S if F is both an almost left bi-quasi ideal of S and an almost right bi-quasi ideal of S [10].

V. A weakly almost left (right) bi-quasi ideal of S if $(wF \cap FwF) \cap F \neq \emptyset$ $\big((Fw \cap FwF) \cap F \neq \emptyset\big)$, for all $w \in S$, and a weakly almost bi-quasi ideal (briefly weakly almost BQ-ideal) of S if F is both a weakly almost left bi-quasi ideal of S and a weakly almost right bi-quasi ideal of S [10].

Definition 7. An almost left (right) bi-quasi ideal A of S is called a minimal almost left (right) bi-quasi ideal of S if for any almost bi-quasi ideal B of S if whenever $B \subseteq A$, then $A = B$ [10].

Definition 8. Let P be an almost bi-quasi ideal of S. Then, P is called

I. A prime almost bi-quasi ideal if for any almost bi-quasi ideals A and B of S such that $AB \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$.

II. A semiprime almost bi-quasi ideal if for any almost bi-quasi ideal A of S such that $AA \subseteq P$ implies that $A \subseteq P$.

III. A strongly prime almost bi-quasi ideal if for any almost bi-quasi ideals A and B of S such that $AB \cap BA \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$ [10].

Definition 9. Let f_S and g_S be soft sets over the common universe U. Then, soft intersection product $f_S \circ g_S$ is defined by [26]

$$(f_S \circ g_S)(x) = \begin{cases} \bigcup_{x=yz} \{f_S(y) \cap g_S(z)\}, & \text{if } \exists y.z \in S \text{ such that } x = yz, \\ \emptyset, & \text{otherwise.} \end{cases}$$

Theorem 1. Let $b_S, d_S, l_S \in S_S(U)$. Then,

I. $(b_S \circ d_S) \circ l_S = b_S \circ (d_S \circ l_S)$.

II. $b_S \circ d_S \neq d_S \circ b_S$, generally.

III. $b_S \circ (d_S \tilde{\cup} l_S) = (b_S \circ d_S) \tilde{\cup} (b_S \circ l_S)$ and $(b_S \tilde{\cup} d_S) \circ l_S = (b_S \circ l_S) \tilde{\cup} (d_S \circ l_S)$.

IV. $b_S \circ (d_S \tilde{\cap} l_S) = (b_S \circ d_S) \tilde{\cap} (b_S \circ l_S)$ and $(b_S \tilde{\cap} d_S) \circ l_S = (b_S \circ l_S) \tilde{\cap} (d_S \circ l_S)$.

V. If $b_S \tilde{\subseteq} d_S$, then $b_S \circ l_S \tilde{\subseteq} d_S \circ l_S$ and $l_S \circ b_S \tilde{\subseteq} l_S \circ d_S$.

VI. If $t_S, k_S \in S_S(U)$ such that $t_S \tilde{\subseteq} b_S$ and $k_S \tilde{\subseteq} d_S$, then $t_S \circ k_S \tilde{\subseteq} b_S \circ d_S$ [26].

Definition 10. Let A be a subset of S. We denote by S_A the soft characteristic function of A and define as

$$S_A(x) = \begin{cases} U, & \text{if } x \in A, \\ \emptyset, & \text{if } x \in S \backslash A. \end{cases}$$

The soft characteristic function of A is a soft set over U, that is, $S_A: S \longrightarrow P(U)$ [26].

If $f_S(x) = U$ for all $x \in S$, then we denote such a kind of soft set by $\tilde{\mathbb{S}}$ throughout this paper. It is obvious that $\tilde{\mathbb{S}} = S_S$, that is, $\tilde{\mathbb{S}}(x) = U$ for all $x \in S$ [26].

Corollary 1. $\mathrm{supp}(S_A) = A$ [46].

Theorem 2. Let X and Y be nonempty subsets of S. Then, the following properties hold [26], [46]:

I. $X \subseteq Y$ if and only if $S_X \tilde{\subseteq} S_Y$.

II. $S_X \tilde{\cap} S_Y = S_{X \cap Y}$ and $S_X \tilde{\cup} S_Y = S_{X \cup Y}$.

III. $S_X \circ S_Y = S_{XY}$.

Definition 11. Let x be an element in S. We denote by S_x the soft characteristic function of x and defined as

$$S_x(y) = \begin{cases} U, & \text{if } y = x, \\ \emptyset, & \text{if } y \neq x. \end{cases}$$

The soft characteristic function of x is a soft set over U, that is, $S_x: S \longrightarrow P(U)$ [47].

Definition 12. A soft set d_S of S over U is called

I. A soft intersection left (right) bi-quasi ideal of S over U if $(\tilde{\mathbb{S}} \circ d_S) \tilde{\cap} (d_S \circ \tilde{\mathbb{S}} \circ d_S) \tilde{\subseteq} d_S$ $\left((d_S \circ \tilde{\mathbb{S}}) \tilde{\cap} (d_S \circ \tilde{\mathbb{S}} \circ d_S) \tilde{\subseteq} d_S\right)$, and a soft intersection bi-quasi ideal of S if d_S is both a soft intersection left bi-quasi ideal of S and a soft intersection right bi-quasi ideal of S [44].

II. A soft intersection almost subsemigroup of S over U if $(d_S \circ d_S) \tilde{\cap} d_S \neq \emptyset_S$ [46].

III. A soft intersection almost left (right) ideal of S over U if $(S_x \circ d_S) \tilde{\cap} d_S \neq \emptyset_S$ $\left((d_S \circ S_x) \tilde{\cap} d_S \neq \emptyset_S\right)$ for all $x \in S$, and a soft intersection almost ideal of S if d_S is both a soft intersection almost left ideal of S and a soft intersection almost right ideal of S [47].

IV. A soft intersection almost bi-ideal of S over U if $(d_S \circ S_x \circ d_S) \tilde{\cap} d_S \neq \emptyset_S$ for all $x \in S$ [48].

V. A soft intersection almost left (right) weak interior ideal of S over U if $(S_x \circ d_S \circ d_S) \tilde{\cap} d_S \neq \emptyset_S$ $\left((d_S \circ d_S \circ S_x) \tilde{\cap} d_S \neq \emptyset_S\right)$ for all $x \in S$, and a soft intersection almost weak interior ideal of S if d_S is both a soft intersection almost left weak interior ideal of S and a soft intersection almost right weak interior ideal of S [49].

VI. A soft intersection almost left (right) tri-ideal of S over U if $(d_S \circ S_x \circ d_S \circ d_S) \tilde{\cap} d_S \neq \emptyset_S$ $\left((d_S \circ d_S \circ S_x \circ d_S) \tilde{\cap} d_S \neq \emptyset_S\right)$, for all $x \in S$, and a soft intersection almost tri-ideal of S if d_S is both a soft intersection almost left tri-ideal of S and a soft intersection almost right tri-ideal of S [50].

VII. A soft intersection almost tri-bi-ideal of S over U if $(d_S \circ d_S \circ S_x \circ d_S \circ d_S) \tilde{\cap} d_S \neq \emptyset_S$ for all $x \in S$ [51].

It is easy to see that if $d_S(x) = U$ for all $x \in S$, then d_S is a soft intersection bi-quasi ideal of S. As is mentioned above, we denote such a kind of soft intersection bi-quasi ideal by $\tilde{\mathbb{S}}$. Regarding the probable consequences of network analysis and graph applications concerning soft sets (defined by the divisibility of determinants), we refer to [52].

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Definition 13. A soft set f_S is called a soft intersection almost left (right) bi-quasi ideal of S if

$$\left[(S_x \circ f_S) \tilde{\cap} \left(f_S \circ S_y \circ f_S\right)\right] \tilde{\cap} f_S \neq \emptyset_S \left(\left[(f_S \circ S_x) \tilde{\cap} \left(f_S \circ S_y \circ f_S\right)\right] \tilde{\cap} f_S \neq \emptyset_S\right)$$

for all $x, y \in S$. f_S is called a soft intersection, the almost bi-quasi ideal of S if f_S is both a soft intersection almost left bi-quasi ideal of S and a soft intersection almost right bi-quasi ideal of S.

A soft set f_S is called a soft intersection weakly almost left (right) bi-quasi ideal of S if

$$[(S_x \circ f_S) \tilde{\cap} (f_S \circ S_x \circ f_S)] \tilde{\cap} f_S \neq \emptyset_S \ ([(f_S \circ S_x) \tilde{\cap} (f_S \circ S_x \circ f_S)] \tilde{\cap} f_S \neq \emptyset_S)$$

for all $x \in S$. f_S is called a soft intersection weakly almost bi-quasi ideal of S if f_S is both a soft intersection weakly almost left bi-quasi ideal of S and a soft intersection weakly almost right bi-quasi ideal of S.

Hereafter, for brevity, a soft intersection is designated by SI, and (left/right) bi-quasi ideal is designed by (left/right) BQ-ideal. Thus, soft intersection (weakly) almost (left/right) bi-quasi ideal is denoted by SI-(weakly) almost (left/right) BQ-ideal.

Here also note that since the operation of soft intersection is commutative in $S_E(U)$, it is obvious that in *Definition 13,* $(S_x \circ f_S)$ and $\left(f_S \circ S_y \circ f_S\right)$ $\left(\text{similarly.} (f_S \circ S_x) \text{ and } \left(f_S \circ S_y \circ f_S\right)\right)$ can commute with each other for all $x. y \in S$.

Example 1. Consider the semigroup $S = \{m. j\}$ under the binary operation with the following table:

Table 1. Cayley table of binary operation.

	m	j
m	m	j
j	j	m

Let f_S, h_S, and g_S be soft sets over $U = D_2 = \{\langle x, y\rangle : x^2 = y^2 = e, xy = yx\} = \{e, x, y, yx\}$ as follows:

$f_S = \{(m. \{e. x\}). (j. \{e\})\}$,

$h_S = \{(m. \{yx\}). (j. \{y. yx\})\}$,

$g_S = \{(m. \{x. y\}). (j. \{e. yx\})\}$.

Here, f_S and h_S are both SI-(weakly) almost BQ-ideal. Let's first show that f_S is an SI-(weakly) almost left BQ-ideal, that is, $\left[(S_x \circ f_S) \tilde{\cap} \left(f_S \circ S_y \circ f_S\right)\right] \tilde{\cap} f_S \neq \emptyset_S$, for all $x, y \in S$.

Let's start with $[(S_m \circ f_S) \tilde{\cap} (f_S \circ S_m \circ f_S)] \tilde{\cap} f_S$:

$$\left[[(S_m \circ f_S) \tilde{\cap} (f_S \circ S_m \circ f_S)] \tilde{\cap} f_S\right](m) = [(S_m \circ f_S) \tilde{\cap} (f_S \circ S_m \circ f_S)](m) \cap f_S(m)$$

$$= (S_m \circ f_S)(m) \cap (f_S \circ S_m \circ f_S)(m) \cap f_S(m)$$

$$= f_S(m) \cap [f_S(m) \cup f_S(j)] \cap f_S(m)$$

$$= f_S(m)$$

$= f_S(m) \cap f_S(j)$.

Consequently,

$$[(S_m \circ f_S) \tilde{\cap} (f_S \circ S_m \circ f_S)] \tilde{\cap} f_S = \{(m.\{e.x\}).(j.\{e\})\} \neq \emptyset_S.$$

Similarly,

$$[(S_m \circ f_S) \tilde{\cap} (f_S \circ S_j \circ f_S)] \tilde{\cap} f_S = \{(m.\{e\}).(j.\{e\})\} \neq \emptyset_S.$$

$$[(S_j \circ f_S) \tilde{\cap} (f_S \circ S_m \circ f_S)] \tilde{\cap} f_S = \{(m.\{e\}).(j.\{e\})\} \neq \emptyset_S.$$

$$[(S_j \circ f_S) \tilde{\cap} (f_S \circ S_j \circ f_S)] \tilde{\cap} f_S = \{(m.\{e\}).(j.\{e\})\} \neq \emptyset_S.$$

Therefore, f_S is an SI-(weakly) almost left BQ-ideal. And also f_S is an SI-(weakly) almost right BQ-ideal, that is, $[(f_S \circ S_x) \tilde{\cap} (f_S \circ S_y \circ f_S)] \tilde{\cap} f_S \neq \emptyset_S$, for all $x, y \in S$. In fact;

$$[(f_S \circ S_m) \tilde{\cap} (f_S \circ S_m \circ f_S)] \tilde{\cap} f_S = \{(m.\{e.x\}).(j.\{e\})\} \neq \emptyset_S.$$

$$[(f_S \circ S_m) \tilde{\cap} (f_S \circ S_j \circ f_S)] \tilde{\cap} f_S = \{(m.\{e\}).(j.\{e\})\} \neq \emptyset_S.$$

$$[(f_S \circ S_j) \tilde{\cap} (f_S \circ S_m \circ f_S)] \tilde{\cap} f_S = \{(m.\{e\}).(j.\{e\})\} \neq \emptyset_S.$$

$$[(f_S \circ S_j) \tilde{\cap} (f_S \circ S_j \circ f_S)] \tilde{\cap} f_S = \{(m.\{e\}).(j.\{e\})\} \neq \emptyset_S,$$

Thus, f_S is an SI-(weakly) almost right BQ-ideal. Hence, f_S is an SI-(weakly) almost BQ-ideal.

Similarly, we can show that h_S is an SI-(weakly) almost left BQ-ideal and SI-(weakly) almost right BQ-ideal. Let's first show that h_S is an SI-(weakly) almost left BQ-ideal:

$$[(S_m \circ h_S) \tilde{\cap} (h_S \circ S_m \circ h_S)] \tilde{\cap} h_S = \{(m.\{yx\}).(j.\{yx\})\} \neq \emptyset_S.$$

$$[(S_m \circ h_S) \tilde{\cap} (h_S \circ S_j \circ h_S)] \tilde{\cap} h_S = \{(m.\{yx\}).(j.\{y.yx\})\} \neq \emptyset_S.$$

$$[(S_j \circ h_S) \tilde{\cap} (h_S \circ S_m \circ h_S)] \tilde{\cap} h_S = \{(m.\{yx\}).(j.\{yx\})\} \neq \emptyset_S.$$

$$[(S_j \circ h_S) \tilde{\cap} (h_S \circ S_j \circ h_S)] \tilde{\cap} h_S = \{(m.\{yx\}).(j.\{yx\})\} \neq \emptyset_S,$$

Consequently, h_S is an SI-(weakly) almost left BQ-ideal. Let's continue with h_S is an SI-(weakly) almost right BQ-ideal:

$$[(h_S \circ S_m) \tilde{\cap} (h_S \circ S_m \circ h_S)] \tilde{\cap} h_S = \{(m.\{yx\}).(j.\{yx\})\} \neq \emptyset_S.$$

$$[(h_S \circ S_m) \tilde{\cap} (h_S \circ S_j \circ h_S)] \tilde{\cap} h_S = \{(m.\{yx\}).(j.\{y.yx\})\} \neq \emptyset_S.$$

$$[(h_S \circ S_j) \tilde{\cap} (h_S \circ S_m \circ h_S)] \tilde{\cap} h_S = \{(m.\{yx\}).(j.\{yx\})\} \neq \emptyset_S.$$

$$[(h_S \circ S_j) \tilde{\cap} (h_S \circ S_j \circ h_S)] \tilde{\cap} h_S = \{(m.\{yx\}).(j.\{yx\})\} \neq \emptyset_S,$$

Therefore, h_S is an SI-(weakly) almost right BQ-ideal. Thus, h_S is an SI-(weakly) almost BQ-ideal.

One can also show that g_S is not an SI-(weakly) almost BQ-ideal. In fact;

$$\left[\left[(S_{j} \circ g_S) \tilde{\cap} (g_S \circ S_{j} \circ g_S)\right] \tilde{\cap} g_S\right](m) = \left[(S_{j} \circ g_S) \tilde{\cap} (g_S \circ S_{j} \circ g_S)\right](m) \cap g_S(m)$$
$$= (S_{j} \circ g_S)(m) \cap (g_S \circ S_{j} \circ g_S)(m) \cap g_S(m)$$
$$= g_S(j) \cap [g_S(m) \cap g_S(j)] \cap g_S(m)$$
$$= g_S(j) \cap g_S(m)$$
$$= \emptyset_S.$$

$$\left[\left[(S_{j} \circ g_S) \tilde{\cap} (g_S \circ S_{j} \circ g_S)\right] \tilde{\cap} g_S\right](j) = \left[(S_{j} \circ g_S) \tilde{\cap} (g_S \circ S_{j} \circ g_S)\right](j) \cap g_S(j)$$
$$= (S_{j} \circ g_S)(j) \cap (g_S \circ S_{j} \circ g_S)(j) \cap g_S(j)$$
$$= g_S(m) \cap [g_S(m) \cup g_S(j)] \cap g_S(j)$$
$$= g_S(m) \cap g_S(j)$$
$$= \emptyset_S.$$

Consequently,

$$\left[(S_{j} \circ g_S) \tilde{\cap} (g_S \circ S_{j} \circ g_S)\right] \tilde{\cap} g_S = \{(m.\emptyset).(j.\emptyset)\} = \emptyset_S.$$

Hence, g_S is not an SI-(weakly) almost left BQ-ideal. Similarly since

$$\left[(g_S \circ S_{j}) \tilde{\cap} (g_S \circ S_{j} \circ g_S)\right] \tilde{\cap} g_{S=}\{(m.\emptyset).(j.\emptyset)\} = \emptyset_S,$$

g_S is not an SI-(weakly) almost right BQ-ideal and g_S is not an SI-(weakly) almost BQ-ideal.

From now on, the proofs are given for only SI-almost left BQ-ideal, since the proofs for SI-almost (right) BQ-ideal can be shown similarly.

Proposition 1. Every SI-almost BQ-ideal is an SI-weakly almost BQ-ideal.

Proof: Let f_S be an SI-almost BQ-ideal. Then,

$\left[(S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S)\right] \tilde{\cap} f_S \neq \emptyset_S$ and $\left[(f_S \circ S_x) \tilde{\cap} (f_S \circ S_y \circ f_S)\right] \tilde{\cap} f_S \neq \emptyset_S$ for all $x.y \in S$. Hence,

$[(S_x \circ f_S) \tilde{\cap} (f_S \circ S_x \circ f_S)] \tilde{\cap} f_S \neq \emptyset_S$ and $[(f_S \circ S_x) \tilde{\cap} (f_S \circ S_x \circ f_S)] \tilde{\cap} f_S \neq \emptyset_S$ for all $x \in S$.

So, f_S is an SI-weakly almost BQ-ideal.

Since SI-weakly almost BQ-ideal is a generalization of SI-almost BQ-ideal, from now on, all the theorems and proofs are given for SI-almost BQ-ideal instead of SI-weakly almost BQ-ideals.

Proposition 2. Let f_S be an SI-left (resp. right) BQ-ideal. f_S is either $(S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S) = \emptyset_S$ $\left((f_S \circ S_x) \tilde{\cap} (f_S \circ S_y \circ f_S) = \emptyset_S\right)$, for some $x, y \in S$ or an SI-almost left (resp. right) BQ-ideal.

Proof: Let f_S be an SI-left BQ-ideal, then, $(\mathbb{S} \circ f_S) \tilde{\cap} (f_S \circ \mathbb{S} \circ f_S) \tilde{\subseteq} f_S$ and let $(S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S) \neq \emptyset_S$. We need to show that

$$\left[(S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S)\right] \tilde{\cap} f_S \neq \emptyset_S,$$

for all $x, y \in S$. Since $(S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S) \tilde{\subseteq} (\mathbb{S} \circ f_S) \tilde{\cap} (f_S \circ \mathbb{S} \circ f_S) \tilde{\subseteq} f_S$, it follows that $(S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S) \tilde{\subseteq} f_S$. From assumption $(S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S) \neq \emptyset_S$ is obvious. Then,

$$\left[(S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S)\right] \tilde{\cap} f_S = (S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S) \neq \emptyset_S,$$

implying that f_S is an SI-almost left BQ-ideal.

Here it is obvious that, if f_S is an SI-left BQ-ideal, and for some $x, y \in S$, $(S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S) = \emptyset_S$ then, $[(S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S)] \tilde{\cap} f_S = \emptyset_S \tilde{\cap} f_S = \emptyset_S$. Therefore, f_S is not an SI-almost left BQ-ideal.

Corollary 2. If f_S is an SI-almost left (resp. right) BQ-ideal, then f_S needs not be an SI-left (resp. right) BQ-ideal.

Example 2. In Example 1, it is shown that h_S is an SI-almost left BQ-ideal; however h_S is not an SI-left bi-quasi ideal. In fact,

$$[(\tilde{\mathbb{S}} \circ h_S) \tilde{\cap} (h_S \circ \tilde{\mathbb{S}} \circ h_S)](m) = (\tilde{\mathbb{S}} \circ h_S)(m) \cap (h_S \circ \tilde{\mathbb{S}} \circ h_S)(m)$$
$$= [h_S(m) \cup h_S(j)] \cap [h_S(m) \cup h_S(j)]$$
$$= h_S(m) \cup h_S(j)$$
$$\not\subseteq h_S(m).$$

Hence, h_S is not an SI-left BQ-ideal. Similarly, h_S is an SI-almost right BQ-ideal; however, h_S is not an SI-right BQ-ideal. In fact,

$$[(h_S \circ \tilde{\mathbb{S}}) \tilde{\cap} (h_S \circ \tilde{\mathbb{S}} \circ h_S)](j) = (h_S \circ \tilde{\mathbb{S}})(j) \cap (h_S \circ \tilde{\mathbb{S}} \circ h_S)(j)$$
$$= [h_S(m) \cup h_S(j)] \cap [h_S(m) \cup h_S(j)]$$
$$= h_S(m) \cup h_S(j)$$
$$\not\subseteq h_S(j).$$

Hence, h_S is not an SI-right BQ-ideal.

Theorem 3. Every SI-almost left (resp. right) BQ-ideal is an SI-almost left (resp. right) ideal.

Proof: Assume that f_S is an SI-almost left BQ-ideal. Hence,

$$[(S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S)] \tilde{\cap} f_S \neq \emptyset_S,$$

for all $x, y \in S$. We need to show that $(S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, for all $x \in S$.

$$[(S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S)] \tilde{\cap} f_S \tilde{\subseteq} (S_x \circ f_S) \tilde{\cap} f_S.$$

Since $[(S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S)] \tilde{\cap} f_S \neq \emptyset_S$, it is obvious that $(S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$. Hence, f_S is an SI-almost left ideal.

Theorem 4. Every SI-almost (left/right) BQ-ideal is an SI-almost bi-ideal.

Proof: Assume that f_S is an SI-almost left BQ-ideal. Hence, $[(f_S \circ S_x \circ f_S) \tilde{\cap} (S_y \circ f_S)] \tilde{\cap} f_S \neq \emptyset_S$, for all $x, y \in S$. We need to show that $(f_S \circ S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, for all $x, y \in S$. $[(f_S \circ S_x \circ f_S) \tilde{\cap} (S_y \circ f_S)] \tilde{\cap} f_S \tilde{\subseteq} (f_S \circ S_x \circ f_S) \tilde{\cap} f_S$ Since $[(f_S \circ S_x \circ f_S) \tilde{\cap} (S_y \circ f_S)] \tilde{\cap} f_S \neq \emptyset_S$, it is obvious that $(f_S \circ S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$. Hence, f_S is an SI-almost bi-ideal.

The following example shows that the converse of Theorem 4 is not true in general:

Example 3. Consider the soft set g_S in Example 1. Here, g_S is an SI-almost bi-ideal, that is, $(g_S \circ S_x \circ g_S) \tilde{\cap} g_S \neq \emptyset_S$, for all $x \in S$. Let's first show that $(g_S \circ S_m \circ g_S) \tilde{\cap} g_S \neq \emptyset_S$:

$$[(g_S \circ S_m \circ g_S) \tilde{\cap} g_S](m) = (g_S \circ S_m \circ g_S)(m) \cap g_S(m)$$
$$= \left[\left(g_S(m) \cap (S_m \circ g_S)(m)\right) \cup \left(g_S(j) \cap (S_m \circ g_S)(j)\right)\right]$$
$$\cap\, g_S(m)$$
$$= [g_S(m) \cup g_S(j)] \cap g_S(m)$$
$$= g_S(m).$$

$$[(g_S \circ S_m \circ g_S) \tilde{\cap} g_S](j) = (g_S \circ S_m \circ g_S)(j) \cap g_S(j)$$
$$= \left[\left(g_S(m) \cap (S_m \circ g_S)(j)\right) \cup \left(g_S(j) \cap (S_m \circ g_S)(m)\right)\right] \cap g_S(j)$$
$$= [g_S(m) \cap g_S(j)] \cap g_S(j)$$
$$= g_S(m) \cap g_S(j).$$

Consequently,

$$(g_S \circ S_m \circ g_S) \tilde{\cap} g_S = \{(m. \{x. y\}). (j. \emptyset)\} \neq \emptyset_S.$$

Similarly,

$$\left(g_S \circ S_j \circ g_S\right) \tilde{\cap} g_S = \{(m. \emptyset). (j. \{e. yx\})\} \neq \emptyset_S.$$

Thus, g_S is an SI-almost bi-ideal. However, it is clear that g_S is not an SI-almost (left/right) BQ-ideal, as seen in Example 1.

Proposition 3. Let f_S be an idempotent soft set. If f_S is an SI-almost (left/right) BQ-ideal, then f_S is an SI-almost subsemigroup.

Proof: Assume that f_S is an SI-almost left BQ-ideal such that f_S is an idempotent, then $f_S \circ f_S = f_S$ and $\left[(S_x \circ f_S) \tilde{\cap} \left(f_S \circ S_y \circ f_S\right)\right] \tilde{\cap} f_S \neq \emptyset_S$, for all $x, y \in S$. We need to show that f_S is an SI-almost subsemigroup, that is $(f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$.

$$\left[(S_x \circ f_S) \tilde{\cap} \left(f_S \circ S_y \circ f_S\right)\right] \tilde{\cap} f_S = \left[(S_x \circ f_S) \tilde{\cap} \left(f_S \circ S_y \circ f_S\right) \tilde{\cap} f_S\right] \tilde{\cap} f_S$$
$$= \left[(S_x \circ f_S) \tilde{\cap} \left(f_S \circ S_y \circ f_S\right) \tilde{\cap} (f_S \circ f_S)\right] \tilde{\cap} f_S$$
$$\tilde{\subseteq} (f_S \circ f_S) \tilde{\cap} f_S.$$

Since $\left[(S_x \circ f_S) \tilde{\cap} \left(f_S \circ S_y \circ f_S\right)\right] \tilde{\cap} f_S \neq \emptyset_S$, it is obvious that $(f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$. Thus, f_S is an SI-almost subsemigroup.

Proposition 4. Let f_S be an idempotent soft set. If f_S is an SI-almost left (resp. right) BQ-ideal, then f_S is an SI-almost left (resp. right) weak interior ideal.

Proof: Assume that f_S is an SI-almost left BQ-ideal such that f_S is an idempotent, then $f_S \circ f_S = f_S$ and $\left[(S_x \circ f_S) \tilde{\cap} \left(f_S \circ S_y \circ f_S\right)\right] \tilde{\cap} f_S \neq \emptyset_S$, for all $x, y \in S$. We need to show that f_S is an SI-almost left weak interior ideal, that is $S_x \circ f_S \circ f_S \tilde{\cap} f_S \neq \emptyset_S$, for all $x \in S$.

$$\left[(S_x \circ f_S) \tilde{\cap} \left(f_S \circ S_y \circ f_S\right)\right] \tilde{\cap} f_S \tilde{\subseteq} (S_x \circ f_S) \tilde{\cap} f_S = (S_x \circ f_S \circ f_S) \tilde{\cap} f_S.$$

Since $\left[(S_x \circ f_S) \tilde{\cap} \left(f_S \circ S_y \circ f_S\right)\right] \tilde{\cap} f_S \neq \emptyset_S$, it is obvious that $(S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$. Thus, f_S is an SI-almost left weak interior ideal.

Proposition 5. Let f_S be an idempotent soft set. If f_S is an SI-almost left (right) BQ-ideal, then f_S is an SI-almost tri-ideal.

Proof: Assume that f_S is an idempotent SI-almost left BQ-ideal such that $f_S \circ f_S = f_S$ and $[(f_S \circ S_x \circ f_S) \tilde{\cap} (S_y \circ f_S)] \tilde{\cap} f_S \neq \emptyset_S$, for all $x, y \in S$. We need to show that f_S is an SI-almost tri-ideal, that is $(f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$ and $(f_S \circ f_S \circ S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, for all $x \in S$.

$$[(f_S \circ S_x \circ f_S) \tilde{\cap} (S_y \circ f_S)] \tilde{\cap} f_S \tilde{\subseteq} (f_S \circ S_x \circ f_S) \tilde{\cap} f_S = (f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S,$$

Since $[(f_S \circ S_x \circ f_S) \tilde{\cap} (S_y \circ f_S)] \tilde{\cap} f_S \neq \emptyset_S$, it is obvious that $(f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$. Hence, f_S is an SI-almost left tri-ideal.

$$[(f_S \circ S_x \circ f_S) \tilde{\cap} (S_y \circ f_S)] \tilde{\cap} f_S \tilde{\subseteq} (f_S \circ S_x \circ f_S) \tilde{\cap} f_S = (f_S \circ f_S \circ S_x \circ f_S) \tilde{\cap} f_S.$$

Since $[(f_S \circ S_x \circ f_S) \tilde{\cap} (S_y \circ f_S)] \tilde{\cap} f_S \neq \emptyset_S$, it is obvious that $(f_S \circ f_S \circ S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$. Hence, f_S is an SI-almost right tri-ideal. Thus, f_S is an SI-almost tri-ideal.

Proposition 6. Let f_S be an idempotent soft set. If f_S is an SI-almost (left/right) BQ-ideal, then f_S is an SI-almost tri-bi-ideal.

Proof: Assume that f_S is an SI-almost left BQ-ideal such that f_S is an idempotent, then $f_S \circ f_S = f_S$ and $[(f_S \circ S_x \circ f_S) \tilde{\cap} (S_y \circ f_S)] \tilde{\cap} f_S \neq \emptyset_S$, for all $x, y \in S$. We need to show that f_S is an SI-almost tri-bi-ideal, that is $(f_S \circ f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, for all $x \in S$.

$$[(f_S \circ S_x \circ f_S) \tilde{\cap} (S_y \circ f_S)] \tilde{\cap} f_S \tilde{\subseteq} (f_S \circ S_x \circ f_S) \tilde{\cap} f_S = (f_S \circ f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S.$$

Since $[(f_S \circ S_x \circ f_S) \tilde{\cap} (S_y \circ f_S)] \tilde{\cap} f_S \neq \emptyset_S$, it is obvious that $(f_S \circ f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$. Thus, f_S is an SI-almost tri-bi-ideal.

Theorem 5. Let $f_S \tilde{\subseteq} h_S$. If f_S is an SI-almost left (resp. right) BQ-ideal, then h_S is an SI-almost left (resp. right) BQ-ideal.

Proof: Assume that f_S is an SI-almost left BQ-ideal. Hence, $[(S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S)] \tilde{\cap} f_S \neq \emptyset_S$, for all $x, y \in S$. We need to show that $[(S_x \circ h_S) \tilde{\cap} (h_S \circ S_y \circ h_S)] \tilde{\cap} h_S \neq \emptyset_S$, for all $x, y \in S$, In fact,

$$[(S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S)] \tilde{\cap} f_S \tilde{\subseteq} [(S_x \circ h_S) \tilde{\cap} (h_S \circ S_y \circ h_S)] \tilde{\cap} h_S \neq \emptyset_S.$$

Since $[(S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S)] \tilde{\cap} f_S \neq \emptyset_S$, it is obvious that $[(S_x \circ h_S) \tilde{\cap} (h_S \circ S_y \circ h_S)] \tilde{\cap} h_S \neq \emptyset_S$. This completes the proof.

Theorem 6. If f_S and h_S are SI-almost left (resp. right) BQ-ideals, then $f_S \tilde{\cup} h_S$ is an SI-almost left (resp. right) BQ-ideal.

Proof: Since f_S is an SI-almost left BQ-ideal and $f_S \tilde{\subseteq} f_S \tilde{\cup} h_S$, $f_S \tilde{\cup} h_S$ is an SI-almost left BQ-ideal by Theorem 5.

Corollary 3. The finite union of SI-almost left (resp. right) BQ-ideals is an SI-almost left (resp. right) BQ-ideal.

Corollary 4. Let f_S or h_S be an SI-almost left (resp. right) BQ-ideal, then $f_S \tilde{\cup} h_S$ is an SI-almost left (resp. right) BQ-ideal.

Here, note that if f_S and h_S are SI-almost left (resp. right) BQ-ideals, then $f_S \tilde{\cap} h_S$ needs not be an SI-almost left (resp. right) BQ-ideal.

Example 4. Consider the SI-almost left (resp. right) BQ-ideal f_S and h_S in Example 1. Since,

$$f_S \tilde{\cap} h_S = \{(m.\emptyset).(j.\emptyset)\} = \emptyset_S,$$

$f_S \tilde{\cap} h_S$ is not an SI-almost left (resp. right) BQ-ideal.

Now, we give the relationship between almost BQ-ideal and SI-almost BQ-ideal. But first of all, we remind the following lemma in order to use it in Theorem 7.

Lemma 1. Let $x \in S$ and Y be a nonempty subset of S. Then, $S_x \circ S_Y = S_{xY}$. If X is a nonempty subset of S and $y \in S$, then $S_X \circ S_y = S_{Xy}$ [47].

Theorem 7. Let A be a nonempty subset of S. Then, A is an almost left (resp. right) BQ-ideal if and only if S_A, the soft characteristic function of A, is an SI-almost left (resp. right) BQ-ideal.

Proof: Assume that $\emptyset \neq A$ is an almost left BQ-ideal. Then,$(xA \cap AyA) \cap A \neq \emptyset$, for all $x, y \in S$, and so there exist $j \in S$ such that $j \in (xA \cap AzA) \cap A$. Since,

$$\left(\left[(S_x \circ S_A) \tilde{\cap} (S_A \circ S_y \circ S_A)\right] \tilde{\cap} S_A\right)(j) = \left((S_{xA} \tilde{\cap} S_{AyA}) \tilde{\cap} S_A\right)(j)$$
$$= \left((S_{xA \cap AyA}) \tilde{\cap} S_A\right)(j)$$
$$= S_{(xA \cap AyA) \cap A}(j)$$
$$= U$$
$$\neq \emptyset.$$

it follows that $\left[(S_x \circ S_A) \tilde{\cap} (S_A \circ S_y \circ S_A)\right] \tilde{\cap} S_A \neq \emptyset_S$. Thus, S_A is an SI-almost left BQ-ideal.

Conversely, assume that S_A is an SI-almost left BQ-ideal. Hence, we have $\left[(S_x \circ S_A) \tilde{\cap} (S_A \circ S_y \circ S_A)\right] \tilde{\cap} S_A \neq \emptyset_S$, for all $x, y \in S$. In order to show that A is an almost left BQ-ideal, we should prove that $A \neq \emptyset$ and $(xA \cap AyA) \cap A \neq \emptyset$, for all $x, y \in S$. $A \neq \emptyset$ is obvious from the assumption. Now,

$$\emptyset_S \neq \left[(S_x \circ S_A) \tilde{\cap} (S_A \circ S_y \circ S_A)\right] \tilde{\cap} S_A$$
$$\Rightarrow \exists s \in S;\ \left(\left[(S_x \circ S_A) \tilde{\cap} (S_A \circ S_y \circ S_A)\right] \tilde{\cap} S_A\right)(s) \neq \emptyset$$
$$\Rightarrow \exists s \in S;\ \left((S_{xA} \tilde{\cap} S_{AyA}) \tilde{\cap} S_A\right)(s) \neq \emptyset$$
$$\Rightarrow \exists s \in S;\ (S_{xA \cap AyA} \tilde{\cap} S_A)(s) \neq \emptyset$$
$$\Rightarrow \exists s \in S;\ S_{(xA \cap AyA) \cap A}(s) = U$$
$$\Rightarrow s \in (xA \cap AyA) \cap A.$$

Hence, $(xA \cap AyA) \cap A \neq \emptyset$. Consequently, A is an almost left BQ-ideal.

Lemma 2. Let $f_S \in S_S(U)$. Then, $f_S \tilde{\subseteq} S_{supp(f_S)}$ [46].

Theorem 8. If f_S is an SI-almost left (resp. right) BQ-ideal, then $supp(f_S)$ is an almost left (resp. right) BQ-ideal.

Proof: Assume that f_S is an SI-almost left BQ-ideal. Thus, $\left[(S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S)\right] \tilde{\cap} f_S \neq \emptyset_S$, for all $x, y \in S$. In order to show that $supp(f_S)$ is an almost left BQ-ideal, by Theorem 8, it is enough to show that $S_{supp(f_S)}$ is an SI-almost left BQ-ideal. By Lemma 2,

$$\left[(S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S)\right] \tilde{\cap} f_S \tilde{\subseteq} \left[(S_x \circ S_{supp(f_S)}) \tilde{\cap} (S_{supp(f_S)} \circ S_y \circ S_{supp(f_S)})\right] \tilde{\cap} S_{supp(f_S)},$$

and $\left[(S_x \circ f_S) \tilde{\cap} (f_S \circ S_y \circ f_S)\right] \tilde{\cap} f_S \neq \emptyset_S$, it implies that
$\left[(S_x \circ S_{supp(f_S)}) \tilde{\cap} (S_{supp(f_S)} \circ S_y \circ S_{supp(f_S)})\right] \tilde{\cap} S_{supp(f_S)} \neq \emptyset_S$. Consequently, $S_{supp(f_S)}$ is an SI-almost left BQ-ideal and by Theorem 7, $supp(f_S)$ is an almost left BQ-ideal.

The following example shows that the converse of Theorem 8 is not true in general:

Example 5. We know that g_S is not an SI-almost left BQ-ideal in Example 1. Since $\mathrm{supp}(g_S) = \{m, j\}$,

$$[(m\{m.j\}) \cap (\{m.j\}m\{m.j\})] \cap \{m.j\} = \{m.j\} \neq \emptyset.$$
$$[(m\{m.j\}) \cap (\{m.j\}j\{m.j\})] \cap \{m.j\} = \{m.j\} \neq \emptyset.$$
$$[(j\{m.j\}) \cap (\{m.j\}m\{e.j\}\{m.j\})] \cap \{m.j\} = \{m.j\} \neq \emptyset.$$
$$[(j\{m.j\}) \cap (\{m.j\}j\{m.j\})] \cap \{m.j\} = \{m.j\} \neq \emptyset.$$

$\mathrm{supp}(g_S)$ is an almost left BQ-ideal. Similarly, g_S is not an SI-almost left BQ-ideal, but since

$$[(\{m.j\}m) \cap (\{m.j\}m\{m.j\})] \cap \{m.j\} = \{m.j\} \neq \emptyset.$$
$$[(\{m.j\}m) \cap (\{m.j\}j\{m.j\})] \cap \{m.j\} = \{m.j\} \neq \emptyset.$$
$$[(\{m.j\}j) \cap (\{m.j\}m\{m.j\})] \cap \{m.j\} = \{m.j\} \neq \emptyset.$$
$$[(\{m.j\}j) \cap (\{m.j\}j\{m.j\})] \cap \{m.j\} = \{m.j\} \neq \emptyset.$$

$\mathrm{supp}(g_S)$ is an almost right BQ-ideal. That is to say, $supp(g_S)$ is an almost BQ-ideal, although g_S is not an SI-almost BQ-ideal.

Definition 14. Let f_S and h_S be SI-almost left (resp. right) BQ-ideals such that $h_S \tilde{\subseteq} f_S$. If $\mathrm{supp}(h_S) = \mathrm{supp}(f_S)$, then f_S is called a minimal SI-almost BQ-ideal.

Theorem 9. Let A be a nonempty subset of S. Then, A is a minimal almost left (resp. right) BQ-ideal if and only if S_A, the soft characteristic function of A, is a minimal SI-almost left (resp. right) BQ-ideal.

Proof: Assume that A is a minimal, almost left BQ-ideal. Thus, A is an almost left BQ-ideal, and so S_A is an SI-almost left BQ-ideal by Theorem 7. Let f_S be an SI-almost left BQ-ideal such that $f_S \tilde{\subseteq} S_A$. By Theorem 8, $\mathrm{supp}(f_S)$ is an almost left BQ-ideal and by Note 1 and Corollary 1,

$$\mathrm{supp}(f_S) \subseteq supp(S_A) = A,$$

Since A is a minimal almost left BQ-ideal, $\mathrm{supp}(f_S) = \mathrm{supp}(S_A) = A$. Thus, S_A is a minimal SI-almost left BQ-ideal by Definition 14.

Conversely, let S_A be a minimal SI-almost left BQ-ideal. Thus, S_A is an SI-almost left BQ-ideal, and A is an almost left BQ-ideal by Theorem 7. Let B be an almost left BQ-ideal such that $B \subseteq A$. By Theorem 7, S_B is an SI-almost left BQ-ideal, and by Theorem 2 (i), $S_B \tilde{\subseteq} S_A$. Since S_A is a minimal SI-almost left BQ-ideal,

$$B = \mathrm{supp}(S_B) = \mathrm{supp}(S_A) = A,$$

by Corollary 1. Thus, A is a minimal, almost left BQ-ideal.

Definition 15. Let f_S, g_S, and h_S be any SI-almost left (resp. right) BQ-ideals. If $h_S \circ g_S \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$ or $g_S \tilde{\subseteq} f_S$, then f_S is called an SI-prime almost left (resp. right) BQ-ideal.

Definition 16. Let f_S and h_S be any SI-almost left (resp. right) BQ-ideals. If $h_S \circ h_S \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$, then f_S is called an SI-semiprime almost left (resp. right) BQ-ideal.

Definition 17. Let f_S, g_S, and h_S be any SI-almost left (resp. right) BQ-ideals. If $(h_S \circ g_S) \tilde{\cap} (g_S \circ h_S) \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$ or $g_S \tilde{\subseteq} f_S$, then f_S is called an SI-strongly prime almost left (resp. right) BQ-ideal.

It is obvious that every SI-strongly prime almost left (resp. right) BQ-ideal is an SI-prime almost left (resp. right) BQ-ideal, and every SI-prime almost left (resp. right) BQ-ideal is an SI-semiprime almost left (resp. right) BQ-ideal.

Theorem 10. If S_P, the soft characteristic function of P, is an SI-prime almost left (resp. right) BQ-ideal, then P is a prime almost left (resp. right) BQ-ideal, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-prime almost left BQ-ideal. Thus, S_P is an SI-almost left BQ-ideal, and thus, P is an almost left BQ-ideal by Theorem 7. Let A and B be almost left BQ-ideals such that $AB \subseteq P$. Thus, by Theorem 7, S_A and S_B are SI-almost left BQ-ideals, and by Theorem 2 (i) and (iii),

$$S_A \circ S_B = S_{AB} \,\widetilde{\subseteq}\, S_P,$$

Since S_P is an SI-prime almost left BQ-ideal and $S_A \circ S_B \,\widetilde{\subseteq}\, S_P$, it follows that $S_A \,\widetilde{\subseteq}\, S_P$ or $S_B \,\widetilde{\subseteq}\, S_P$. Therefore, by Theorem 2 (i), $A \subseteq P$ or $B \subseteq P$. Consequently, P is a prime almost left BQ-ideal.

Theorem 11. If S_P, the soft characteristic function of P, is an SI-semiprime almost left (resp. right) BQ-ideal, then P is a semiprime almost left (resp. right) BQ-ideal, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-semiprime almost left BQ-ideal. Thus, S_P is an SI-almost left BQ-ideal, and thus, P is an almost left BQ-ideal by Theorem 7. Let A be an almost left BQ-ideal such that $AA \subseteq P$. Thus, by Theorem 7, S_A is an SI-almost left BQ-ideal, and by Theorem 2 (i) and (ii),

$$S_A \circ S_A = S_{AA} \,\widetilde{\subseteq}\, S_P.$$

Since S_P is an SI-prime almost left BQ-ideal and $S_A \circ S_A \,\widetilde{\subseteq}\, S_P$, it follows that $S_A \,\widetilde{\subseteq}\, S_P$. Therefore, by Theorem 2 (i) $A \subseteq P$. Consequently, P is a semiprime almost left BQ-ideal.

Theorem 12. If S_P, the soft characteristic function of P, is an SI-strongly prime almost left (resp. right) BQ-ideal, then P is a strongly prime almost left (resp. right) BQ-ideal, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-strongly prime almost left BQ-ideal. Thus, S_P is an SI-almost left BQ-ideal, and thus, P is an almost left BQ-ideal by Theorem 7. Let A and B be almost left BQ-ideals such that $AB \cap BA \subseteq P$. Thus, by Theorem 7, S_A and S_B are SI-almost left BQ-ideals, and by Theorem 2,

$$(S_A \circ S_B) \,\widetilde{\cap}\, (S_B \circ S_A) = S_{AB} \,\widetilde{\cap}\, S_{BA} = S_{AB\cap BA} \,\widetilde{\subseteq}\, S_P.$$

Since S_P is an SI-strongly prime almost left BQ-ideal and $(S_A \circ S_B) \,\widetilde{\cap}\, (S_B \circ S_A) \,\widetilde{\subseteq}\, S_P$, it follows that $S_A \,\widetilde{\subseteq}\, S_P$ or $S_B \,\widetilde{\subseteq}\, S_P$. Thus, by Theorem 2 (i), $A \subseteq P$ or $B \subseteq P$. Therefore, P is a strongly prime, almost left BQ-ideal.

4 | Conclusion

The concepts of "soft intersection almost bi-quasi ideal" and "soft intersection weakly almost bi-quasi ideal" of semigroups were defined in this work. We demonstrated that although any soft intersection almost bi-quasi ideal is also a soft intersection weakly almost bi-quasi ideal, a soft intersection almost ideal, and a soft intersection almost bi-ideal of a semigroup; the converses are not true in general with counterexamples. Additionally, it was shown that an idempotent soft intersection almost bi-quasi ideal is a soft intersection almost subsemigroup, a soft intersection almost weak interior ideal, a soft intersection almost tri-ideal, and a soft intersection almost tri-bi-ideal. We obtained the relation between soft intersection almost bi-quasi ideal of a semigroup and almost bi-quasi ideal of a semigroup according to minimality, primeness, semiprimeness, and strongly primeness with the obtained theorem that if a nonempty set A is almost bi-quasi ideal then its soft characteristic function is soft intersection almost bi-quasi ideal, and vice versa. Additionally, we investigated that, unlike soft intersection operation, soft union operation can form a semigroup with the collection of almost bi-quasi ideals of a semigroup. In future studies, some kinds of semigroup ideals, such as quasi-interior ideals, bi-interior ideals, and bi-quasi-interior ideals, may be studied in terms of soft intersection almost ideals.

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