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Some Characterization of Fermatean Fuzzy $\mathcal{L}$-ring Ideals

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Abstract Abstract

The Fermatean fuzzy set (FFS) represents a robust approach for addressing ambiguity, effectively managing issues that remain unresolved by Intuitionistic fuzzy set and Pythagorean fuzzy set concepts. Due to its practical utility and significant impact on tackling real-world challenges across various domains, FFS has spurred extensive research. This study defines Fermatean fuzzy sublattice and Fermatean fuzzy lattice. Additionally, it introduces Fermatean fuzzy $\mathcal{L}$-ring ideals. The paper explores the concept of homomorphism within Fermatean fuzzy sets. Furthermore, it investigates important findings concerning the image and pre-image of Fermatean fuzzy $\mathcal{L}$-ring ideals, utilizing properties of infimum and supremum. The results are illustrated through pertinent numerical examples.

Keywords: Intuitionistic fuzzy sets, Pythagorean fuzzy sets, Fermatean fuzzy sets, Fermatean fuzzy lattice, Fermatean fuzzy $\mathcal{L}$-ring ideal.

1|Introduction

Fuzzy sets (FSs), introduced by Zadeh [33] to model imprecise evaluations, have paved the way for intuitionistic fuzzy sets (IFSs). Atanassov [4] extended FSs by introducing membership and non-membership degrees, with the constraint that their sum does not exceed 1. The characterization of intuitionistic fuzzy sets as a generalized

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form of fuzzy sets is notably intriguing and practical across diverse application domains. Atanassov [5] further advanced this concept by introducing innovative operations defined over intuitionistic fuzzy sets.

Some situations where $\mu + \nu \geq 1$ exists, unlike in IFSs. Because of this necessity in IFSs, Pythagorean fuzzy sets were developed (PFSs). The Pythagorean fuzzy set (PFS), design in [31, 30] was a unique tool for dealing with ambiguity when evaluating membership grade and nonmembership grade fulfilling the constraints $0 \leq \mu \leq 1$ and $0 \leq \nu \leq 1$, with the result that $\mu^2 + \nu^2 \leq 1$. PFSs were better at characterizing unclear data than IFSs.

Fermatean fuzzy sets was the extension Pythagorean fuzzy sets. In Fermatean fuzzy sets the membership grade (μ) and non-membership grade (ν) satisfy the conditions $0 \leq \mu^3 + \nu^3 \leq 1$, where the values of μ and ν lie between 0 and 1. Senapati et al. [25] defines some new operations over Fermatean fuzzy numbers. Fermatean fuzzy sets were applied in wide range of fields, including artificial intelligence, signal processing, management sciences, engineering, mathematics, social sciences, automata theory, medical sciences, biology and statistics etc.

Algebraic patterns find application in theoretical physics, information science, computer science, control engineering, and numerous other domains. This diversity drives scholars to explore various abstract algebra topics and discoveries within the broader framework of fuzzy set theories. Kunchman et al. [18] introduced the concept of fuzzy prime ideals in gamma near rings. Kim and Jun [15, 16] pioneered the intuitionistic fuzzification of multiple semigroup ideals. Kim and Lee [17] explored the implications of intuitionistic fuzzy bi-ideals in semigroups. Sardar et al. [23] introduced prime ideals, semi-prime ideals, and Γ-semigroups with intuitionistic fuzzy information. Adak et al. [**?**, **?**, **?**] investigated properties of Pythagorean fuzzy ideals in near-rings and presented results on rough Pythagorean fuzzy sets.

The lattice structure holds significant prominence in fuzzy set theory, extensively discussed and applied across various contexts. R. Natarajan and S. Moganavalli [20] extended fuzzy sets into lattice theory. Ajmal et al. [2] explored the lattice of fuzzy rings. Various authors have introduced and investigated fuzzy lattices in distinct manners [27, 28]. Similar approaches have been taken in the realm of intuitionistic fuzzy lattices. Thomas et al. introduced intuitionistic fuzzy lattices by fuzzifying membership elements within lattice structures and examined their properties. Tripathy et al. [29] proposed intuitionistic fuzzy lattices based on intuitionistic fuzzy order relations introduced by Burillo and Bustine [7, 8].

Furthermore, K. Hur, Y. S. Ahn, and D. S. Kim [13] expanded the lattice of intuitionistic fuzzy sets in rings. Marashdeh and Salleh [**?**] introduced and studied intuitionistic fuzzy ideals based on intuitionistic $\mathcal{L}$-fuzzy sets. Sasireka et al. [24] established properties of intuitionistic fuzzy sub $\mathcal{L}$-rings. G. J. Wang [32] generalized order-homomorphisms on fuzzy sets. Olson [21] provided proofs on homomorphisms for hemirings. Palaniappan [22] introduced and studied homomorphisms and anti-homomorphisms of fuzzy and anti-fuzzy ideals.

This paper introduces the concept of Fermatean fuzzy sublattice and lattice, and extends it to define Fermatean fuzzy $\mathcal{L}$-ideals, exploring their properties. Additionally, it presents insightful characterizations of Fermatean fuzzy $\mathcal{L}$-ring ideals and investigates homomorphisms of these ideals. The study demonstrates that the image and pre-image under homomorphisms of Fermatean fuzzy $\mathcal{L}$-ring ideals remain Fermatean fuzzy $\mathcal{L}$-ring ideals.

The paper is structured as follows: Section 2 provides preliminary definitions and examples. In Section 3, significant results concerning Fermatean fuzzy $\mathcal{L}$-ring ideals and their homomorphisms are discussed. Section 4 concludes the paper.

2|Preliminaries and Definition

We will go over the ideas that are connected to fuzzy sets, intuitionistic fuzzy sets, Pythagorean fuzzy sets and Fermatean fuzzy sets in this section.

Definition 1. *A poset $(L, \leq)$ is said to form a lattice of for any $a, b \in L$,* $\sup\{a, b\}$ *and* $\inf\{a, b\}$ *exist in L.*

In this case we write $\sup\{a, b\} = a \vee b$ *and* $\inf\{a, b\} = a \wedge b$.

Throughout this paper we denote a lattice with join '$\vee$' and '$\wedge$' by simply L.

Definition 2. *We define the fuzzy set $\mathcal{M}$ within a universal set S as*

$$\mathcal{M} = \{\langle \xi, \alpha_{\mathcal{M}}(\xi) \rangle : \xi \in S\},$$

where $\alpha_{\mathcal{M}} : S \to [0,1]$ is a mapping known as the fuzzy membership function.

The complement of α is defined by $\bar{\alpha}(\xi) = 1 - \alpha(\xi)$ for all $\xi \in S$ and is denoted by $\bar{\alpha}$.

Definition 3. *An intuitionistic fuzzy set (IFS) $\mathcal{I}$ in S is defined as*

$$\mathcal{I} = \{\langle \xi, \alpha_{\mathcal{I}}(\xi), \beta_{\mathcal{I}}(\xi)\rangle : \xi \in S\},$$

where the $\alpha_{\mathcal{I}}(\xi)$ is the worth of membership and $\beta_{\mathcal{I}}(\xi)$ is the worth of non-membership of the element $\xi \in S$ respectively.

Also $\alpha_{\mathcal{I}} : S \to [0,1], \beta_{\mathcal{I}} : S \to [0,1]$ and satisfy the condition

$$0 \leq \alpha_{\mathcal{I}}(\xi) + \beta_{\mathcal{I}}(\xi) \leq 1,$$

for all $\xi \in S$.

The degree of indeterminacy $h_{\mathcal{I}}(\xi) = 1 - \alpha_{\mathcal{I}}(\xi) - \beta_{\mathcal{I}}(\xi)$.

In some circumferences $0 \leq \alpha_{\mathcal{I}}(\xi) + \beta_{\mathcal{I}}(\xi) \leq 1$ for whatever reason, this may not be the case. We take some situations where $0.9 + 0.4 = 1.3 > 1$, but $0.9^2 + 0.4^2 = 0.97 < 1$. To deal with this problem,, Yager [31, 30] proposed the perspective assumes of Pythagorean fuzzy set in 2013.

Definition 4. *A Pythagorean fuzzy set $\mathcal{P}$ in universe of discourse S is represented as*

$$\mathcal{P} = \{\langle \xi, \alpha_{\mathcal{P}}(\xi), \beta_{\mathcal{P}}(\xi)\rangle | \xi \in S\},$$

where $\alpha_{\mathcal{P}}(\xi) : S \to [0,1]$ denotes the worth of membership and $\beta_{\mathcal{P}}(\xi) : S \to [0,1]$ represents the worth to which the element $\xi \in S$ is not a member of the set $\mathcal{P}$, with the condition that

$$0 \leq (\alpha_{\mathcal{P}}(\xi))^2 + (\beta_{\mathcal{P}}(\xi))^2 \leq 1,$$

for all $\xi \in S$.

The worth of indeterminacy $h_{\mathcal{P}}(\xi) = \sqrt{1 - (\alpha_{\mathcal{P}}(\xi))^2 - (\beta_{\mathcal{P}}(\xi))^2}$.

In practice, the condition $0 \leq \alpha_{\mathcal{P}}^2(\xi) + \beta_{\mathcal{P}}^2(\xi) \leq 1$ may not be true for any reason. For example $0.9^2 + 0.5^2 = 1.06 > 1$, but $0.9^3 + 0.5^3 = 0.854 < 1$, or $0.8^2 + 0.7^2 = 1.13 > 1$, but $0.8^3 + 0.7^3 = .855 < 1$. To address this issue, Senapati et. al., [25] proposed the principle of the fermatean fuzzy set in 2021.

Definition 5. *A fermatean fuzzy set $\mathcal{M}$ in a finite universe of discourse S is furnished as*

$$\mathcal{M} = \{\langle \xi, \alpha_{\mathcal{M}}(\xi), \beta_{\mathcal{M}}(\xi)\rangle | \xi \in S\},$$

where $\alpha_{\mathcal{M}}(\xi) : S \to [0,1]$ denotes the worth of membership and $\beta_{\mathcal{M}}(\xi) : S \to [0,1]$ represents the worth to which the element $\xi \in S$ is not a member of the set $\mathcal{M}$, with the predicament that

$$0 \leq (\alpha_{\mathcal{M}}(\xi))^3 + (\beta_{\mathcal{M}}(\xi))^3 \leq 1,$$

for all $\xi \in S$.

The worth of indeterminacy $h_{\mathcal{M}}(\xi) = \sqrt[3]{1 - (\alpha_{\mathcal{M}}(\xi))^3 - (\beta_{\mathcal{M}}(\xi))^3}$.

Definition 6. *A fuzzy subset α of a lattice ordered ring (or $\mathcal{L}$-ring in short) R, is called fuzzy sub $\mathcal{L}$-ring of R, if the following conditions are satisfied.*
(i) $\alpha(\xi \vee \eta) \geq \min\{\alpha(\xi), \alpha(\eta)\}$
(ii) $\alpha(\xi \wedge \eta) \geq \min\{\alpha(\xi), \alpha(\eta)\}$
(iii) $\alpha(\xi - \eta) \geq \min\{\alpha(\xi), \alpha(\eta)\}$
(iv) $\alpha(\xi\eta) \geq \min\{\alpha(\xi), \alpha(\eta)\}\ \forall\ \xi, \eta \in L$.

Definition 7. *A fuzzy subset α of an $\mathcal{L}$-ring R, is called fuzzy $\mathcal{L}$-ring ideal (or) fuzzy $\mathcal{L}$-ideals of R, if the following conditions are satisfied.*
(i) $\alpha(\xi \vee \eta) \geq \min\{\alpha(\xi), \alpha(\eta)\}$
(ii) $\alpha(\xi \wedge \eta) \geq \max\{\alpha(\xi), \alpha(\eta)\}$

(iii) $\alpha(\xi - \eta) \geq \min\{\alpha(\xi), \alpha(\eta)\}$
(iv) $\alpha(\xi\eta) \geq \max\{\alpha(\xi), \alpha(\eta)\}\ \forall\, \xi, \eta \in \mathcal{L}$.

Definition 8. *A mapping ϕ from ring R to a ring S is called on Homomorphism, if $\forall\, \xi, \eta \in R$*
(i) $\phi(\xi + \eta) = \phi(\xi) + \phi(\eta)$
(ii) $\phi(\xi\eta) = \phi(\xi) \cdot \phi(\eta)$.

Definition 9. [20] *Let R and S be two $\mathcal{L}$-rings. A function $\phi : R \to S$ is called an $\mathcal{L}$-Homomorphism if for all $\xi, \eta \in R$*
(i) $\phi(\xi \vee \eta) = \phi(\xi) \vee \phi(\eta)$
(ii) $\phi(\xi \wedge \eta) = \phi(\xi) \wedge \phi(\eta)$
(iii) $\phi(\xi + \eta) = \phi(\xi) + \phi(\eta)$
(iv) $\phi(\xi\eta) = \phi(\xi) \cdot \phi(\eta)$.

Definition 10. [?] *Let ϕ be a mapping from a set R to a set S and let A be a fuzzy subset in R. Then A is called ϕ-invarient if $\phi(\xi) = \phi(\eta)$ implies $A(\xi) = A(\eta)$ for all $\xi, \eta \in R$. Clearly, if A is ϕ-invarient, then $\phi^{-1}(\phi(A)) = A$*

Definition 11. *A Fermatean fuzzy set $\mathcal{M}$ is said to have sup-property [Inf-property], if for each subset $T \subset A$ there exist $t_0 \in T$ such that,*

$$\sup_{t \in T}\{\alpha(t)\} = \alpha(t_0)$$
$$\inf_{t \in T}\{\beta(t)\} = \beta(t_0)$$

Definition 12. *Let ϕ be a mapping from a set X to a set Y and let $\langle \alpha_{\mathcal{M}}, \beta_{\mathcal{M}} \rangle$ be Fermatean fuzzy subset in X and Y respectively.*
(i) $\phi(A)$, the image of A under ϕ, is a intuition fuzzy subset in Y for all $\eta \in Y$. we define,

$$\phi(\alpha_{\mathcal{M}})(\eta) = \begin{cases} \sup\limits_{\xi \in \phi^{-1}(\eta)} \alpha_{\mathcal{M}}(\xi), & \text{if } \phi^{-1}(\eta) \neq \phi \\ 0 & \text{if } \phi^{-1}(\eta) = \phi \end{cases}$$

and

$$\phi(\beta_{\mathcal{M}})(\eta) = \begin{cases} \inf\limits_{\xi \in \phi^{-1}(\eta)} \beta_{\mathcal{M}}(\xi), & \text{if } \phi^{-1}(\eta) \neq \phi \\ 1 & \text{if } \phi^{-1}(\eta) = \phi \end{cases}$$

$\phi^{-1}(B)$ is the pre-image of B under ϕ, is a Fermatean fuzzy set in X.
i.e,

$$\phi^{-1}(\alpha_{\mathcal{M}})(\xi) = \alpha_{\mathcal{M}}(\phi(\xi))$$

and

$$\phi^{-1}(\beta_{\mathcal{M}})(\xi) = \beta_{\mathcal{M}}(\phi(\xi)), \forall\, \xi \in R.$$

Definition 13. *Let $\mathcal{L}$ be a lattice and $\mathcal{M} = \{\langle S, \alpha_{\mathcal{M}}(\xi), \beta_{\mathcal{M}}(\xi) \rangle : \xi \in S\}$ be Fermatean fuzzy set of $\mathcal{L}$. Then $\mathcal{M}$ is called an Fermatean fuzzy sublattice (Fermatean fuzzy lattice) of $\mathcal{L}$ if the following conditions are satisfied.*
(i) $\alpha_{\mathcal{M}}(\xi \vee \eta) \geq \min\{\alpha_{\mathcal{M}}(\xi), \alpha_{\mathcal{M}}(\eta)\}$
(ii) $\alpha_{\mathcal{M}}(\xi \wedge \eta) \geq \min\{\alpha_{\mathcal{M}}(\xi), \alpha_{\mathcal{M}}(\eta)\}$
(iii) $\beta_{\mathcal{M}}(\xi \vee \eta) \leq \max\{\beta_{\mathcal{M}}(\xi), \beta_{\mathcal{M}}(\eta)\}$
(iv) $\beta_{\mathcal{M}}(\xi \wedge \eta) \leq \max\{\beta_{\mathcal{M}}(\xi), \beta_{\mathcal{M}}(\eta)\}\ \forall\, \xi, \eta \in \mathcal{L}$.

Definition 14. *A Fermatean fuzzy sublattice $\mathcal{M}$ of $\mathcal{L}$ is called an Fermatean fuzzy ideal of $\mathcal{L}$ (Fermatean fuzzy $\mathcal{L}$-ideal) if the following conditions are satisfied*
(i) $\alpha(\xi \vee \eta) \geq \min\{\alpha(\xi), \alpha(\eta)\}$
(ii) $\alpha(\xi \wedge \eta) \geq \max\{\alpha(\xi), \alpha(\eta)\}$
(iii) $\beta(\xi \vee \eta) \leq \max\{\beta(\xi), \beta(\eta)\}$
(iv) $\beta(\xi \wedge \eta) \leq \min\{\beta(\xi), \beta(\eta)\}\ \forall\, \xi, \eta \in \mathcal{L}$.

Definition 15. *Let R be a ring. A Fermatean fuzzy set $\mathcal{M} = \{\langle \xi, \alpha(\xi), \beta(\xi)\rangle : \xi \in R\}$ of R is said to be Fermatean fuzzy sub $\mathcal{L}$-ring on R if for all $\xi, \eta \in R$,*
(i) $\alpha(\xi - \eta) \geq \min\{\alpha(\xi), \alpha(\eta)\}$
(ii) $\alpha(\xi\eta) \geq \min\{\alpha(\xi), \alpha(\eta)\}$
(iii) $\alpha(\xi \vee \eta) \geq \min\{\alpha(\xi), \alpha(\eta)\}$
(iv) $\alpha(\xi \wedge \eta) \geq \min\{\alpha(\xi), \alpha(\eta)\}$
(v) $\beta(\xi - \eta) \leq \max\{\beta(\xi), \beta(\eta)\}$
(vi) $\beta(\xi\eta) \leq \max\{\beta(\xi), \beta(\eta)\}$
(vii) $\beta(\xi \vee \eta) \leq \max\{\beta(\xi), \beta(\eta)\}$
(viii) $\beta(\xi \wedge \eta) \leq \max\{\beta(\xi), \beta(\eta)\}$.

3|Main Results

In this section, we define Fermatean fuzzy $\mathcal{L}$-ring ideal and defined Fermatean fuzzy homomorphism from a $\mathcal{L}$-ring R into $\mathcal{L}$-ring S. Also, investigated some related results on image and preimage of homomorphism of Fermatean fuzzy $\mathcal{L}$-ring utilizing supremum and infimum property of Fermatean fuzzy sets.

Definition 16. *A Fermatean fuzzy sub $\mathcal{L}$-ring $\mathcal{M}$ on R is said to be a Fermatean fuzzy left $\mathcal{L}$-ring ideal if for all $\xi, \eta \in R$.*
(i) $\alpha(\xi - \eta) \geq \min\{\alpha(\xi), \alpha(\eta)\}$
(ii) $\alpha(\xi\eta) \geq \alpha(\eta)$
(iii) $\alpha(\xi \vee \eta) \geq \min\{\alpha(\xi), \alpha(\eta)\}$
(iv) $\alpha(\xi \wedge \eta) \geq \max\{\alpha(\xi), \alpha(\eta)\}$
(v) $\beta(\xi - \eta) \leq \max\{\beta(\xi), \beta(\eta)\}$
(vi) $\beta(\xi\eta) \leq \beta(\eta)$
(vii) $\beta(\xi \vee \eta) \leq \max\{\beta(\xi), \beta(\eta)\}$
(viii) $\beta(\xi \wedge \eta) \leq \min\{\beta(\xi), \beta(\eta)\}$.

Definition 17. *A Fermatean fuzzy sub $\mathcal{L}$-ring $\mathcal{M}$ on R is said to be a Fermatean fuzzy right $\mathcal{L}$- ring ideal if for all $\xi, \eta \in R$.*
(i) $\alpha(\xi - \eta) \geq \min\{\alpha(\xi), \alpha(\eta)\}$
(ii) $\alpha(\xi\eta) \geq \alpha(\xi)$
(iii) $\alpha(\xi \vee \eta) \geq \min\{\alpha(\xi), \alpha(\eta)\}$
(iv) $\alpha(\xi \wedge \eta) \geq \max\{\alpha(\xi), \alpha(\eta)\}$
(v) $\beta(\xi - \eta) \leq \max\{\beta(\xi), \beta(\eta)\}$
(vi) $\beta(\xi\eta) \leq \beta(\xi)$
(vii) $\beta(\xi \vee \eta) \leq \max\{\beta(\xi), \beta(\eta)\}$
(viii) $\beta(\xi \wedge \eta) \leq \min\{\beta(\xi), \beta(\eta)\}$.

Definition 18. *A Fermatean fuzzy sub $\mathcal{L}$-ring $\mathcal{M}$ on R is said to be a Fermatean fuzzy $\mathcal{L}$-ring ideal if it is both an Fermatean fuzzy left $\mathcal{L}$- ring ideal and an Fermatean fuzzy right $\mathcal{L}$- ring ideal of R.*

Example 1. *Now $(R\{= \{\xi_1, \xi_2, \xi_3, \xi_4\}, +, \cdot, \vee, \wedge)$ is a $\mathcal{L}$-ring under the operations $+$, $\cdot$, $\vee$, and $\wedge$ defined by the following tables,*

$+$	ξ_1	ξ_2	ξ_3	ξ_4
ξ_1	ξ_1	ξ_2	ξ_3	ξ_4
ξ_2	ξ_2	ξ_1	ξ_4	ξ_3
ξ_3	ξ_3	ξ_4	ξ_1	ξ_2
ξ_4	ξ_4	ξ_3	ξ_2	ξ_1

$\cdot$	ξ_1	ξ_2	ξ_3	ξ_4
ξ_1	ξ_1	ξ_1	ξ_1	ξ_1
ξ_2	ξ_1	ξ_2	ξ_1	ξ_2
ξ_3	ξ_1	ξ_1	ξ_3	ξ_3
ξ_4	ξ_1	ξ_2	ξ_3	ξ_4

$\vee$	ξ_1	ξ_2	ξ_3	ξ_4
ξ_1	ξ_1	ξ_2	ξ_3	ξ_4
ξ_2	ξ_2	ξ_2	ξ_4	ξ_4
ξ_3	ξ_3	ξ_4	ξ_3	ξ_4
ξ_4	ξ_4	ξ_4	ξ_4	ξ_4

$\wedge$	ξ_1	ξ_2	ξ_3	ξ_4
ξ_1	ξ_1	ξ_1	ξ_1	ξ_1
ξ_2	ξ_1	ξ_2	ξ_1	ξ_2
ξ_3	ξ_1	ξ_1	ξ_3	ξ_3
ξ_4	ξ_1	ξ_2	ξ_3	ξ_4

Consider

$$\alpha(\xi) = \begin{cases} 0.8 & \text{if } \xi = \xi_1 \\ 0.6 & \text{if } \xi = \xi_2 \\ 0.3 & \text{if } \xi = \xi_3, \xi_4, \end{cases} \qquad \beta(\xi) = \begin{cases} 0.1 & \text{if } \xi = \xi_1 \\ 0.2 & \text{if } \xi = \xi_2 \\ 0.4 & \text{if } \xi = \xi_3, \xi_4 \end{cases}$$

then $(R\{= \{\xi_1, \xi_2, \xi_3, \xi_4\}, +, \cdot, \vee, \wedge)$ *is a Fermatean fuzzy* $\mathcal{L}$*-ring ideal.*

Theorem 1. *Let* ϕ *be a homomorphism from a* $\mathcal{L}$*-ring* R *into* $\mathcal{L}$*-ring* S *and let* $\langle \alpha_{\mathcal{M}}, \beta_{\mathcal{M}} \rangle$ *be a Fermatean fuzzy left* $\mathcal{L}$*- ring ideal of* S*. Then the pre-image* $\langle \phi^{-1}(\alpha_{\mathcal{M}}), \phi^{-1}(\eta_{\mathcal{M}}) \rangle$ *is a Fermatean fuzzy left* $\mathcal{L}$*- ring ideal of* R*.*

Proof. Consider a $\mathcal{L}$-ring Homomorphism $\phi : R \to S$.
Let $\langle \alpha_{\mathcal{M}}, \beta_{\mathcal{M}} \rangle$ be a Fermatean fuzzy left $\mathcal{L}$- ring ideal of S. $\forall\ \xi,\ \eta \in R$.

$$\begin{aligned} \text{(i) } \phi^{-1}(\alpha_{\mathcal{M}})(\xi - \eta) &= \alpha_{\mathcal{M}}\phi(\xi - \eta) \\ &\geq \min\{\alpha_{\mathcal{M}}\phi(\xi), \alpha_{\mathcal{M}}\phi(\eta)\} \\ \phi^{-1}(\alpha_{\mathcal{M}})(\xi - \eta) &\geq \min\{\phi^{-1}(\alpha_{\mathcal{M}})(\xi), \phi^{-1}(\alpha_{\mathcal{M}})(\eta)\}. \end{aligned}$$

$$\begin{aligned} \text{(ii) } \phi^{-1}(\alpha_{\mathcal{M}})(\xi\eta) &= \alpha_{\mathcal{M}}\phi(\xi\eta) \\ &\geq \alpha_{\mathcal{M}}(\phi(\eta) \\ &\geq \phi^{-1}(\alpha_{\mathcal{M}}\phi(\eta)) \\ \phi^{-1}(\alpha_{\mathcal{M}})(\xi\eta) &\geq \phi^{-1}(\alpha_{\mathcal{M}}(\eta)\}. \end{aligned}$$

$$\begin{aligned} \text{(iii) } \phi^{-1}(\alpha_{\mathcal{M}})(\xi \vee \eta) &= \alpha_{\mathcal{M}}\phi(\xi \vee \eta) \\ &\geq \min\{\alpha_{\mathcal{M}}\phi(\xi), \alpha_{\mathcal{M}}\phi(\eta)\} \\ \phi^{-1}(\alpha_{\mathcal{M}})(\xi \vee \eta) &\geq \min\{\phi^{-1}(\alpha_{\mathcal{M}})(\xi), \phi^{-1}(\alpha_{\mathcal{M}})(\eta)\}. \end{aligned}$$

$$\begin{aligned} \text{(iv) } \phi^{-1}(\alpha_{\mathcal{M}})(\xi \wedge \eta) &= \alpha_{\mathcal{M}}\phi(\xi \wedge \eta) \\ &\geq \max\{\alpha_{\mathcal{M}}\phi(\xi), \alpha_{\mathcal{M}}\phi(\eta)\} \\ \phi^{-1}(\alpha_{\mathcal{M}})(\xi \vee \eta) &\geq \max\{\phi^{-1}(\alpha_{\mathcal{M}})(\xi), \phi^{-1}(\alpha_{\mathcal{M}})(\eta)\}. \end{aligned}$$

$$\begin{aligned} \text{(v) } \phi^{-1}(\beta_{\mathcal{M}})(\xi - \eta) &= \beta_{\mathcal{M}}\phi(\xi - \eta) \\ &\leq \max\{\beta_{\mathcal{M}}\phi(\xi), \beta_{\mathcal{M}}\phi(\eta)\} \\ \phi^{-1}(\beta_{\mathcal{M}})(\xi - \eta) &\leq \max\{\phi^{-1}(\beta_{\mathcal{M}})(\xi), \phi^{-1}(\beta_{\mathcal{M}})(\eta)\}. \end{aligned}$$

$$\begin{aligned} \text{(vi) } \phi^{-1}(\beta_{\mathcal{M}})(\xi\eta) &= \beta_{\mathcal{M}}\phi(\xi\eta) \\ &\leq \beta_{\mathcal{M}}(\phi(\eta) \\ &\leq \phi^{-1}(\beta_{\mathcal{M}}\phi(\eta)) \\ \phi^{-1}(\alpha_{\mathcal{M}})(\xi\eta) &\geq \phi^{-1}(\beta_{\mathcal{M}}(\eta)\}. \end{aligned}$$

$$\begin{aligned} \text{(vii) } \phi^{-1}(\beta_{\mathcal{M}})(\xi \vee \eta) &= \beta_{\mathcal{M}}\phi(\xi \vee \eta) \\ &\leq \max\{\beta_{\mathcal{M}}\phi(\xi), \beta_{\mathcal{M}}\phi(\eta)\} \\ \phi^{-1}(\beta_{\mathcal{M}})(\xi \vee \eta) &\leq \max\{\phi^{-1}(\beta_{\mathcal{M}})(\xi), \phi^{-1}(\beta_{\mathcal{M}})(\eta)\}. \end{aligned}$$

$$\begin{aligned} \text{(viii) } \phi^{-1}(\beta_{\mathcal{M}})(\xi \wedge \eta) &= \beta_{\mathcal{M}}\phi(\xi \wedge \eta) \\ &\leq \min\{\beta_{\mathcal{M}}\phi(\xi), \beta_{\mathcal{M}}\phi(\eta)\} \\ \phi^{-1}(\beta_{\mathcal{M}})(\xi \vee \eta) &\leq \min\{\phi^{-1}(\beta_{\mathcal{M}})(\xi), \phi^{-1}(\beta_{\mathcal{M}})(\eta)\}. \end{aligned}$$

$\therefore\ \phi^{-1}(\alpha_{\mathcal{M}}), \phi^{-1}(\beta_{\mathcal{M}})$ is a Fermatean fuzzy left $\mathcal{L}$- ring ideal of R.

Theorem 2. *Let* $\phi : R \to S$ *be a homomorphism from a* $\mathcal{L}$*-ring* R *into* S*. If* $\alpha_{\mathcal{M}},\ \beta_{\mathcal{M}}$ *is a Fermatean fuzzy right* $\mathcal{L}$*- ring ideal of* S*, then pre-image* $\phi^{-1}(\alpha_{\mathcal{M}})$ *and* $\phi^{-1}(\beta_{\mathcal{M}})$ *is a Fermatean fuzzy right* $\mathcal{L}$*- ring ideal of* R*.*

Proof. Consider homomorphism $\phi : R \to S$, where R and S be $\mathcal{L}$-ring. Let $\alpha_{\mathcal{M}}, \beta_{\mathcal{M}}$ be a Fermatean fuzzy right $\mathcal{L}$-ring ideal of $S\ \forall\ \xi,\ \eta \in R$.

$$\begin{aligned} \text{(i) } \phi^{-1}(\alpha_{\mathcal{M}})(\xi - \eta) &= \alpha_{\mathcal{M}}\phi(\xi - \eta) \\ &\geq \min\{\alpha_{\mathcal{M}}\phi(\xi), \alpha_{\mathcal{M}}\phi(\eta)\} \\ \phi^{-1}(\alpha_{\mathcal{M}})(\xi - \eta) &\geq \min\{\phi^{-1}(\alpha_{\mathcal{M}})(\xi), \phi^{-1}(\alpha_{\mathcal{M}})(\eta)\}. \end{aligned}$$

$$\begin{aligned} \text{(ii) } \phi^{-1}(\alpha_{\mathcal{M}})(\xi\eta) &= \alpha_{\mathcal{M}}(\phi(\xi\eta)) \\ &\geq \alpha_{\mathcal{M}}(\phi(\xi) \\ &= \phi^{-1}(\alpha_{\mathcal{M}})(\xi))\phi^{-1}(\alpha_{\mathcal{M}})(\xi\eta) \\ &\geq \phi^{-1}((\alpha_{\mathcal{M}})(\xi)\}. \end{aligned}$$

$$\begin{aligned} \text{(iii) } \phi^{-1}(\alpha_{\mathcal{M}})(\xi \vee \eta) &= \alpha_{\mathcal{M}}\phi(\xi \vee \eta) \\ &\geq \min\{\alpha_{\mathcal{M}}\phi(\xi), \alpha_{\mathcal{M}}\phi(\eta)\} \\ \phi^{-1}(\alpha_{\mathcal{M}})(\xi \vee \eta) &\geq \min\{\phi^{-1}(\alpha_{\mathcal{M}})(\xi), \phi^{-1}(\alpha_{\mathcal{M}})(\eta)\}. \end{aligned}$$

$$\begin{aligned} \text{(iv) } \phi^{-1}(\alpha_{\mathcal{M}})(\xi \wedge \eta) &= \alpha_{\mathcal{M}}(\phi(\xi \wedge \eta)) \\ &\geq \max\{\alpha_{\mathcal{M}}\phi(\xi), \alpha_{\mathcal{M}}\phi(\eta)\} \\ \phi^{-1}(\alpha_{\mathcal{M}})(\xi \vee \eta) &\geq \max\{\phi^{-1}(\alpha_{\mathcal{M}})(\xi), \phi^{-1}(\alpha_{\mathcal{M}})(\eta)\}. \end{aligned}$$

$$\begin{aligned} \text{(v) } \phi^{-1}(\beta_{\mathcal{M}})(\xi - \eta) &= \beta_{\mathcal{M}}\phi(\xi - \eta) \\ &\leq \max\{\beta_{\mathcal{M}}\phi(\xi), \beta_{\mathcal{M}}\phi(\eta)\} \\ \phi^{-1}(\beta_{\mathcal{M}})(\xi - \eta) &\leq \max\{\phi^{-1}(\beta_{\mathcal{M}})(\xi), \phi^{-1}(\beta_{\mathcal{M}})(\eta)\}. \end{aligned}$$

$$\begin{aligned} \text{(vi) } \phi^{-1}(\beta_{\mathcal{M}})(\xi\eta) &= \beta_{\mathcal{M}}(\phi(\xi\eta)) \\ &\leq \beta_{\mathcal{M}}(\phi(\xi) \\ &\leq \phi^{-1}(\beta_{\mathcal{M}}\phi(\xi)) \\ \phi^{-1}(\alpha_{\mathcal{M}})(\xi\eta) &\geq \phi^{-1}(\beta_{\mathcal{M}}(\eta)\}. \end{aligned}$$

$$\begin{aligned} \text{(vii) } \phi^{-1}(\beta_{\mathcal{M}})(\xi \vee \eta) &= \beta_{\mathcal{M}}\phi(\xi \vee \eta) \\ &\leq \max\{\beta_{\mathcal{M}}\phi(\xi), \beta_{\mathcal{M}}\phi(\eta)\} \\ \phi^{-1}(\beta_{\mathcal{M}})(\xi \vee \eta) &\leq \max\{\phi^{-1}((\beta_{\mathcal{M}})(\xi)), \phi^{-1}(\beta_{\mathcal{M}}(\eta))\}. \end{aligned}$$

$$\begin{aligned} \text{(viii) } \phi^{-1}(\beta_{\mathcal{M}})(\xi \wedge \eta) &= \beta_{\mathcal{M}}\phi(\xi \wedge \eta) \\ &\leq \min\{\beta_{\mathcal{M}}\phi(\xi), \beta_{\mathcal{M}}\phi(\eta)\} \\ \phi^{-1}(\beta_{\mathcal{M}})(\xi \vee \eta) &\leq \min\{\phi^{-1}(\beta_{\mathcal{M}}(\xi)), \phi^{-1}(\beta_{\mathcal{M}}(\eta))\}. \end{aligned}$$

$\therefore\ \phi^{-1}(\alpha_{\mathcal{M}}), \phi^{-1}(\beta_{\mathcal{M}})$ is a Fermatean fuzzy right $\mathcal{L}$- ring ideal of R.

Theorem 3. *Let $\phi : R \to S$ be a homomorphism from a $\mathcal{L}$-ring R into $\mathcal{L}$-ring S. If $\alpha_{\mathcal{M}},\ \beta_{\mathcal{M}}$ is a Fermatean fuzzy $\mathcal{L}$- ring ideal of S, then pre-image $\phi^{-1}(\alpha_{\mathcal{M}}),\ \phi^{-1}(\beta_{\mathcal{M}})$ is a Fermatean fuzzy right $\mathcal{L}$- ring ideal of R.*

Proof. It's trivial.

Theorem 4. *Let R and S be $\mathcal{L}$-ring and $\phi : R \to S$ be a homomorphism. If $\alpha_{\mathcal{M}},\ \beta_{\mathcal{M}}$ is a Fermatean fuzzy left $\mathcal{L}$- ring ideal of a $\mathcal{L}$-ring R, with sup property, then the pre-image $\phi(\alpha_{\mathcal{M}})$ and $\phi(\beta_{\mathcal{M}})$ is a Fermatean fuzzy left $\mathcal{L}$-ring ideal of S.*

Proof. Consider homomorphism $\phi : R \to S$, where R and S be $\mathcal{L}$-ring. Let $\alpha_{\mathcal{M}}, \beta_{\mathcal{M}}$ be a Fermatean fuzzy left $\mathcal{L}$- ring ideal of $R \,\forall\, \xi,\ \eta \in R$.

$$\begin{aligned}
\text{(i) } \phi(\alpha_{\mathcal{M}})(\phi(\xi)-\phi(\eta)) &= \phi(\alpha_{\mathcal{M}})\phi(\xi-\eta)\\
&\geq \min\{\alpha_{\mathcal{M}}(\xi), \alpha_{\mathcal{M}}(\eta)\}\\
\phi(\alpha_{\mathcal{M}})(\phi(\xi)-\phi(\eta)) &\geq \min\{\phi(\alpha_{\mathcal{M}})(\xi), \phi(\alpha_{\mathcal{M}})(\eta)\}.
\end{aligned}$$

$$\begin{aligned}
\text{(ii) } \phi(\alpha_{\mathcal{M}})(\phi(\xi)\phi(\eta)) &= \phi(\alpha_{\mathcal{M}})(\phi(\xi\eta))\\
&\geq \alpha_{\mathcal{M}}(\eta)\\
&= \phi(\alpha_{\mathcal{M}})\phi(\eta))\\
\phi(\alpha_{\mathcal{M}})(\phi(\xi)\phi(\eta)) &\geq \phi(\alpha_{\mathcal{M}})(\phi(\eta).
\end{aligned}$$

$$\begin{aligned}
\text{(iii) } \phi(\alpha_{\mathcal{M}})(\phi(\xi)\vee\phi(\eta)) &= \phi(\alpha_{\mathcal{M}})\phi(\xi\vee\eta)\\
&\geq \min\{\alpha_{\mathcal{M}}(\xi), \alpha_{\mathcal{M}}(\eta)\}\\
\phi(\alpha_{\mathcal{M}})(\phi(\xi)\vee\phi(\eta)) &\geq \min\{\phi(\alpha_{\mathcal{M}})(\xi), \phi(\alpha_{\mathcal{M}})(\eta)\}.
\end{aligned}$$

$$\begin{aligned}
\text{(iv) } \phi(\alpha_{\mathcal{M}})(\phi(\xi)\wedge\phi(\eta)) &= \phi(\alpha_{\mathcal{M}})(\phi(\xi\wedge\eta))\\
&\geq \max\{\alpha_{\mathcal{M}}(\xi), \alpha_{\mathcal{M}}(\eta)\}\\
\phi(\alpha_{\mathcal{M}})(\phi(\xi)\wedge\phi(\eta)) &\geq \max\{\phi(\alpha_{\mathcal{M}})(\xi), \phi(\alpha_{\mathcal{M}})(\eta)\}.
\end{aligned}$$

$$\begin{aligned}
\text{(v) } \phi(\beta_{\mathcal{M}})(\phi(\xi)-\phi(\eta)) &= \phi(\beta_{\mathcal{M}})\phi(\xi-\eta)\\
&\leq \max\{\beta_{\mathcal{M}}(\xi), \beta_{\mathcal{M}}(\eta)\}\\
\phi(\beta_{\mathcal{M}})(\phi(\xi)-\phi(\eta)) &\leq \max\{\phi(\beta_{\mathcal{M}})(\xi), \phi(\beta_{\mathcal{M}})(\eta)\}.
\end{aligned}$$

$$\begin{aligned}
\text{(vi) } \phi(\beta_{\mathcal{M}})(\phi(\xi)\phi(\eta)) &= \phi(\beta_{\mathcal{M}})\phi(\xi\eta)\\
&\leq \beta_{\mathcal{M}}(\phi(\eta))\\
&\leq \phi(\beta_{\mathcal{M}}\phi(\eta))\\
\phi(\beta_{\mathcal{M}})(\phi(\xi)\phi(\eta)) &\geq \phi(\beta_{\mathcal{M}})(\eta)).
\end{aligned}$$

$$\begin{aligned}
\text{(vii) } \phi(\beta_{\mathcal{M}})(\phi(\xi)\vee\phi(\eta)) &= \phi(\beta_{\mathcal{M}})\phi(\xi\vee\eta)\\
&\leq \max\{\beta_{\mathcal{M}}(\xi), \beta_{\mathcal{M}}(\eta)\}\\
\phi(\beta_{\mathcal{M}})(\phi(\xi)\vee\phi(\eta)) &\leq \max\{\phi(\beta_{\mathcal{M}})(\xi)), \phi(\beta_{\mathcal{M}})(\eta)\}.
\end{aligned}$$

$$\begin{aligned}
\text{(viii) } \phi(\beta_{\mathcal{M}})(\phi(\xi)\wedge\phi(\eta)) &= \phi(\beta_{\mathcal{M}})(\phi(\xi\wedge\eta))\\
&\leq \min\{\beta_{\mathcal{M}}(\xi), \beta_{\mathcal{M}}(\eta)\}\\
\phi(\beta_{\mathcal{M}})(\phi(\xi)\wedge\phi(\eta)) &\leq \min\{\phi(\beta_{\mathcal{M}})(\xi), \phi(\beta_{\mathcal{M}})(\eta)\}.
\end{aligned}$$

$\therefore\ \phi(\alpha_{\mathcal{M}}), \phi(\beta_{\mathcal{M}})$ is a Fermatean fuzzy right $\mathcal{L}$- ring ideal of S.

Theorem 5. *Let $\phi : R \to S$ be a homomorphism from a $\mathcal{L}$-ring R into $\mathcal{L}$-ring S. If $\alpha_{\mathcal{M}},\ \beta_{\mathcal{M}}$ is a Fermatean fuzzy right $\mathcal{L}$- ring ideal of R, with sup property, then the pre-image $\phi(\alpha_{\mathcal{M}})$ and $\phi(\beta_{\mathcal{M}})$ is a Fermatean fuzzy right $\mathcal{L}$-ring ideal of S.*

Proof. Consider a $\mathcal{L}$-ring homomorphism $\phi : R \to S$.
Let $\alpha_{\mathcal{M}}, \beta_{\mathcal{M}}$ be a Fermatean fuzzy right $\mathcal{L}$- ring ideal of $R \,\forall\, \xi,\ \eta \in R$.

$$\begin{aligned}
\text{(i) } \phi(\alpha_{\mathcal{M}})(\phi(\xi)-\phi(\eta)) &= \phi(\alpha_{\mathcal{M}})\phi(\xi-\eta)\\
&\geq \min\{\alpha_{\mathcal{M}}(\xi), \alpha_{\mathcal{M}}(\eta)\}\\
\phi(\alpha_{\mathcal{M}})(\phi(\xi)-\phi(\eta)) &\geq \min\{\phi(\alpha_{\mathcal{M}})(\xi), \phi(\alpha_{\mathcal{M}})(\eta)\}.
\end{aligned}$$

$$\begin{aligned}
\text{(ii) } \phi(\alpha_{\mathcal{M}})(\phi(\xi)\phi(\eta)) &= \phi(\alpha_{\mathcal{M}})(\phi(\xi\eta)) \\
&\geq \alpha_{\mathcal{M}}(\xi) \\
&= \phi(\alpha_{\mathcal{M}})\phi(\xi)) \\
\phi(\alpha_{\mathcal{M}})(\phi(\xi)\phi(\eta)) &\geq \phi(\alpha_{\mathcal{M}})(\phi(\xi).
\end{aligned}$$

$$\begin{aligned}
\text{(iii) } \phi(\alpha_{\mathcal{M}})(\phi(\xi) \vee \phi(\eta)) &= \phi(\alpha_{\mathcal{M}})\phi(\xi \vee \eta) \\
&\geq \min\{\alpha_{\mathcal{M}}(\xi), \alpha_{\mathcal{M}}(\eta)\} \\
\phi(\alpha_{\mathcal{M}})(\phi(\xi) \vee \phi(\eta)) &\geq \min\{\phi(\alpha_{\mathcal{M}})(\xi), \phi(\alpha_{\mathcal{M}})(\eta)\}.
\end{aligned}$$

$$\begin{aligned}
\text{(iv) } \phi(\alpha_{\mathcal{M}})(\phi(\xi) \wedge \phi(\eta)) &= \phi(\alpha_{\mathcal{M}})(\phi(\xi \wedge \eta)) \\
&\geq \max\{\alpha_{\mathcal{M}}(\xi), \alpha_{\mathcal{M}}(\eta)\} \\
\phi(\alpha_{\mathcal{M}})(\phi(\xi) \wedge \phi(\eta)) &\geq \max\{\phi(\alpha_{\mathcal{M}})(\xi), \phi(\alpha_{\mathcal{M}})(\eta)\}.
\end{aligned}$$

$$\begin{aligned}
\text{(v) } \phi(\beta_{\mathcal{M}})(\phi(\xi) - \phi(\eta)) &= \phi(\beta_{\mathcal{M}})\phi(\xi - \eta) \\
&\leq \max\{\beta_{\mathcal{M}}(\xi), \beta_{\mathcal{M}}(\eta)\} \\
\phi(\beta_{\mathcal{M}})(\phi(\xi) - \phi(\eta)) &\leq \max\{\phi(\beta_{\mathcal{M}})(\xi), \phi(\beta_{\mathcal{M}})(\eta)\}.
\end{aligned}$$

$$\begin{aligned}
\text{(vi) } \phi(\beta_{\mathcal{M}})(\phi(\xi)\phi(\eta)) &= \phi(\beta_{\mathcal{M}})\phi(\xi\eta) \\
&\leq \beta_{\mathcal{M}}(\phi(\xi)) \\
&\leq \phi(\beta_{\mathcal{M}}(\xi)) \\
\phi(\beta_{\mathcal{M}})(\phi(\xi)\phi(\eta)) &\geq \phi(\beta_{\mathcal{M}})(\phi(\xi)).
\end{aligned}$$

$$\begin{aligned}
\text{(vii) } \phi(\beta_{\mathcal{M}})(\phi(\xi) \vee \phi(\eta)) &= \phi(\beta_{\mathcal{M}})\phi(\xi \vee \eta) \\
&\leq \max\{\beta_{\mathcal{M}}(\xi), \beta_{\mathcal{M}}(\eta)\} \\
\phi(\beta_{\mathcal{M}})(\phi(\xi) \vee \phi(\eta)) &\leq \max\{\phi(\beta_{\mathcal{M}})(\xi), \phi(\beta_{\mathcal{M}})(\eta)\}.
\end{aligned}$$

$$\begin{aligned}
\text{(viii) } \phi(\beta_{\mathcal{M}})(\phi(\xi) \wedge \phi(\eta)) &= \phi(\beta_{\mathcal{M}})(\phi(\xi \wedge \eta)) \\
&\leq \min\{\beta_{\mathcal{M}}(\xi), \beta_{\mathcal{M}}(\eta)\} \\
\phi(\beta_{\mathcal{M}})(\phi(\xi) \wedge \phi(\eta)) &\leq \min\{\phi(\beta_{\mathcal{M}})(\xi), \phi(\beta_{\mathcal{M}})(\eta)\}.
\end{aligned}$$

$\therefore$ $\phi(\alpha_{\mathcal{M}}), \phi(\beta_{\mathcal{M}})$ is a Fermatean fuzzy right $\mathcal{L}$- ring ideal of S.

Theorem 6. *Let $\phi : R \to S$ be homomorphism from $\mathcal{L}$-ring R into $\mathcal{L}$-ring S. If $\alpha_{\mathcal{M}}$, $\beta_{\mathcal{M}}$ is an Fermatean fuzzy $\mathcal{L}$- ring ideal of a $\mathcal{L}$-ring R with sup property, then pre-image $\phi(\alpha_{\mathcal{M}})$, $\phi(\beta_{\mathcal{M}})$ is a fuzzy $\mathcal{L}$- ring ideal of $\mathcal{L}$-ring S.*

Proof. It's trivial.

4|Conclusion

Within this detailed study, we define lattice concept of Fermatean fuzzy sets along with their crucial results in the Fermatean fuzzy sets context. The left and right ideals of Fermatean fuzzy sets over lattice is initiated. The concept of homomorphism on Fermatean fuzzy $\mathcal{L}$-ring ideal are introduced. With the help of supremum and infimum property of Fermatean fuzzy sets, some important results related to image and pre-image of homomorphism of Fermatean fuzzy $\mathcal{L}$-ring ideal has been investigated. To extend this work, one can discuss some characterization on anti-homomorphism properties of Fermatean fuzzy $\mathcal{L}$-ring ideal.

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Data Availability

All data supporting the reported findings in this research paper are provided within the manuscript.

Conflicts of Interest

The authors declare that there is no conflict of interest concerning the reported research findings.

References

[1] A. K. Adak, Interval-Valued Intuitionistic Fuzzy Subnear Rings, *Handbook of Research on Emerging Applications of Fuzzy Algebraic Structures* IGI-Global, (2020) 213-224.
[2] N. Ajmal, and K. V. Thomas, The Lattice of Fuzzy ideals of a ring. *Fuzzy sets and Systems*, 371-379, (1995).
[3] N. Ajmal, Homomorphism of fuzzy Subgroups, Correspondents theorem, *Fuzzy Sets and Systems*, 61(1994) 329-339.
[4] K. T. Atanassov, Intuitionistic fuzzy sets. *Fuzzy sets and Systems*, 20(1) (1986), 87-96.
[5] K. T. Atanassov, New operations defined over the intuitionistic fuzzy sets. *Fuzzy sets and Systems*, 61, (1994), 137-142.
[6] G. Brikhoff, Lattice Theory, *Published by American Mathematical Theory Providence* , Rhode, Island, (1967).
[7] P. Burillo and H. Bustince, Intuitionistic fuzzy relations (Part I), *Mathware and computing*, 2: 5–38, 1995.
[8] P. Burillo and H. Bustince, Intuitionistic fuzzy relations (Part II), Effect of Atanassov's operators on the properties of the bi fuzzy relations, *Mathware and computing*, 2:117–148, 1995.
[9] K. Chandrasekharan Rao and Swaminathan, Anti-Homomorphism in Near Rings, *Jr of Inst. of maths and computer sciences (Math.Ser)*, Vol.2 (2006), 83-88.
[10] Fang Jin-Xuan, Fuzzy Homomorphism and Fuzzy isomorphism,*Fuzzy Sets and Systems*, 63(1994), 237-242.
[11] A. A. M. Hassan, On Fuzzy Rings and Fuzzy Homomorphisms, *The Journal of fuzzy Mathematics*, Vol.7, No.2, 1999.
[12] K. Hur, Y. S. Ahn and DS. Kim The Lattice of Intuitionistic Fuzzy Ideals of a Ring, *Journal of Appl.math and Computing* , 18(12), (2005) No, 12pp, 465-486.
[13] K. Hur, S. Y. Jang and H. W. Kang, Intuitionistic Fuzzy Ideals of a Ring, *J. Korea Soc Math. Educ Ser. B: Pure Appl. Math.*, 12(3),(2005), 193-209.
[14] K. Hur, H. W. Kang, and H. K. Song, Intuitionistic Fuzzy Sub- groups and Subrings,*Honam Math J.* 25 (1), (2003), 19-41.
[15] K. H. Kim and Y. B. Jun, Intuitionistic fuzzy ideals of semigroups, *Indian J. Pure Appl. Math.* 33(4) (2002) 443-449.
[16] K. H. Kim and Y.B. Jun, Intuitionistic fuzzy interior ideals of semigroups, *Int. J. Math. Math. Sci.* 27 (5) (2001) 261-267.
[17] K. H. Kim and J. G. Lee, On fuzzy bi-ideals of semigroups, *Turk. J. Math.* 29 (2005) 201–210.
[18] S. P. Kuncham, S. Bhavanari, Fuzzy prime ideal of a gamma near ring. *Soochow Journal of Mathematics* 31 (1) (2005) 121-129.
[19] K. Meena, and K. V. Thomas, Intuitionistic L-fuzzy Subrings, *International Mathematical Forum*, 6(52), (2011), 2561-2572.
[20] R. Natrajan, S. Moganavalli, Fuzzy Sublattice Ordered Rings, *International journals of contemporary Mathematical Sciences*, 7(13), (2012), 625-630.
[21] D. M. Olson, On the Homomorphism for Hemirings, *IJMMS*, 1(1978), 439-445.
[22] Palaniappan.N and K.Arjunan, The Homomorphism Anti- Homomorphism of a Fuzzy and Anti-Fuzzy ideals, Varahmihir journal of mathematical sciences, vol.6 no.1(2006), 181-188.
[23] S. K. Sardar, S. K. Majumder and M. Mandal, Atanassov's intuitionistic fuzzy ideals of Γ-semigroups, *Int. J. Algebra* 5(7) (2011) 335-353.
[24] KR. Sasireka, KE. Sathappan and B. Chellapa Intuitionistic Fuzzy l- filters, *International Journal of Mathematics and its applications*, 4(2c), (2016), 171-178.
[25] T. Senapati, T. and R. R. Yager, Some new operations over Fermatean fuzzy numbers and application of Fermatean fuzzy WPM in multiple criteria decision making, *Informatica*, 30(2) 391-412, (2019).
[26] A. Sheikabdullah and K. Jeyaraman, Anti-homomorphism in fuzzy ideals of rings, *Int. J. Contemp. Math. Sciences*, 5(55), (2010), 2717-2721.
[27] K. V. Thomas and L. S. Nair, Intuitionistic fuzzy sublattices and ideals, *Fuzzy Information and Engineering*, 3(2011), 321–331.
[28] K. V. Thomas and L. S. Nair, Rough intuitionistic fuzzy sets in a lattice, *International Mathematical Forum*, 6(2011), 1327–1335.
[29] B. K. Tripathy, M. K. Satapathy and P. K. Choudhury, Intuitionistic fuzzy lattices and intuitionistic fuzzy Boolean algebras, *International Journal of Engineering and Technology*, 5(2013), 2352–2361.

[30] R. R. Yager, Abbasov AM. Pythagorean membeship grades, complex numbers and decision making. *Int J Intell Syst* 28 (2013) 436-452.

[31] R. R. Yager, Pythagorean fuzzy subsets. *In: Proc Joint IFSA World Congress and NAFIPS Annual Meeting*, Edmonton, Canada, (2013) 57-61.

[32] G. J. Wang, Order-Homomorphism on fuzzy sets, *Fuzzy sets and Systems* , 12 (1984), 281-288.

[33] L. A. Zadeh, Fuzzy Sets, *Inform and Control*, 8(1965) 338-353.